

PMP

PRISON MATHEMATICS
PROJECT

THE PRISON MATHEMATICS PROJECT NEWSLETTER

SPRING 2024 – ITERATION 8

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Welcome to the latest iteration of the Prison Mathematics Project newsletter. There's something in these pages for everyone—from the merely math-curious to those pondering the most intriguing and complex mysteries of the universe.

Our writers, editors and contributors have designed this newsletter to share with you our deep appreciation for the wonders waiting to be discovered through the study and exploration of mathematics. We connect prisoners who are dedicated to change with mentors who share their passion for mathematics. In this way we provide an essential framework for rebuilding their lives during their incarceration.

Together we enter a world that transcends walls, where all are welcome and anyone who wishes can embrace the beauty, the humanity and the transformative power of mathematics.

You can become an active participant in your own rehabilitation. We invite you to experience a new culture that leads to desistance from crime and meaningful employment opportunities post-incarceration, but more than that, to human flourishing. Wherever you are on your own journey into the realm of mathematics, we're here for you.

ASK THE MATH GURU

Do numbers freak you out? Does math send you into meltdown-mode? Are you wondering why we need to learn this stuff ANYways?? Don't worry, The Math Guru is here to help you work through your math trauma, one problem at a time. Ask for advice, guidance, or just a good ol' pep talk! You got this!



Dear Math Guru Vanessa,

I have a procrastination problem. I am not lazy, but when I look at a problem I feel intimidated and inadequate to complete it.

Over the years I have learned to break large problems into parts. Sometimes that works. At other times it makes it worse, because even the individual parts seem intimidating.

I also tend to be down on myself. I'm not smart enough, studied enough or competent enough. It seems easier just to put it off.

Being in prison does not help. With all the ways the system works on the psyche, it seems to reinforce my feelings of inadequacy.

I try to be "proactive" and not let outside things influence my thinking. This is easier said than done. It seems the harder I try, the more voices there are in my head telling me I can't do it.

Working with a mentor from PMP has helped my confidence level. But I still have to battle my way through feeling intimidated and inadequate. Then I get down on myself for being down on myself. Seems pretty silly when I write this down. . .

How do I get those lies out of my mind? Is there a way to structure an action plan so it is less intimidating rather than more? How do I accept my limits while not disparaging myself?

I hope you can help. I suspect this is too much for one short column, but thanks for your time!

Robert Noll
PMP Participant



Vanessa Vakharia,
The Math Guru

Robert! First of all, I have to say thank you SO MUCH for being brave enough to write this letter. I can guarantee you that there are SO many others that feel like you, and you writing this is going to help them as well! The truth is that ALL of us have limiting beliefs, and as an expert on math anxiety, I can tell you that so many of our limiting beliefs come from the incorrect assumption that we will never have the skills or just aren't smart enough to handle math. But everyone deals with these incorrect assumptions, so you're not alone! Your strategy of breaking things down into bite-sized tasks is a really, really good one, so congratulations for taking that step!

My main piece of advice is honestly to go easy on yourself :) We all get down on ourselves. We all have highs and lows. The trick is to remember that that negative voice in your head telling you that you're not good enough isn't your ONLY voice, and it's not the truth. Our brains have what we call a "negativity bias," which means that we tend to always think the worst case scenario, no matter what. Just knowing that can help you tell that voice to "chill out" when it comes on too strong!

Just remember, you are not your thoughts. You have overcome incredible obstacles in your life; math is just another challenge for you to take on. Have fun, celebrate your successes, and learn from your failures. You are tackling a subject most people are WAY too scared to even look at.

You rock!



Vanessa Vakharia

Got math anxiety? Think you're a hopeless case? Vanessa to the rescue! The Lady Gaga of mathematics will put her Master's in Mathematics Education to good use by delivering the ultimate personalized pep-talk!

✉ vanessa@themathguru.ca

📍 Prison Mathematics Project,
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Dear Math Guru Vanessa,

I have a daughter who is into math, she is studying algebra in high school right now, and I don't even have a GED. Even though I have had little schooling, I want to be able to connect with her through her interest in algebra as well as encourage her in this direction. Is there any way of doing this without full-on studying math? Are there any books or any other materials that I can get that we can both use and understand that focus on early high school mathematics?

Jeremiah (Jay) Jackson

Death Row, ODRC Chillicothe CI

Jeremiah, thank you so much for reaching out and thank you so much for sharing your experience! Interestingly, what you're experiencing is what MOST parents experience when their kids are in high school—seriously! GED or not, MOST parents are totally uncomfortable with high school math and also have anxiety around math from their own negative experiences! As a result, many feel like they can't connect with their kids. One thing I suggest to parents is to focus less on

the actual math content (algebra, equations, etc) and more on the skills needed to solve math problems. For example, if your daughter is struggling in school, it's likely because she's feeling fear around making mistakes, or feels like she's not smart enough, that kind of thing. I'm sure you have years of experience with feeling those things—as adults, we *all* do! Maybe you can come up with some advice for her when she's feeling like she wants to give up? Or perhaps a pep talk when she feels like a math problem is impossible to solve or understand? That will allow you to help and connect with her regardless of how much algebra you know!

One other thing I suggest to parents who want to connect is to ask your daughter to explain the math TO you! Teaching math TO you can help her learn the material in a deeper way, and you might actually learn something interesting! Your daughter is super lucky to have a dad like you, and I'm excited for you to connect further and happy that you had the courage to write this letter. I'm sure many others are experiencing this as well and your letter will help them too!



Vanessa Vakharia

ANNOUNCEMENTS

There's a lot to be excited about as we work to spread the joy of mathematics to all ♥

- You can now add the PMP on Corrlinks! pmp@pmathp.org
- Now available on Securus: MATH THERAPY, a weekly podcast by Vanessa Vakharia, our very own Ask the Math Guru columnist.

To listen to Math Therapy:

1. Open the podcast app on your Securus device
2. Select "Add Podcast" from the dropdown menu
3. Click "Search" and enter the name "Math Therapy."
4. Enjoy!

- A paper two years in the making, co-authored by PMP co-founder Christopher Havens and Carsten Elsner from the University of Applied Sciences in Hannover, Germany, has been published in the journal *Integers*.

The paper is titled: "*On natural leaping convergents of regular continued fractions and an application to linear fractional transformations*"

Here are the reference data for those with access to the internet:

Integers, volume 23 (2023), A82.

<http://math.colgate.edu/~integers/x82/x82.pdf>

This finally fulfills Christopher's wish for a systematic treatment of linear fractional transformations of continued fractions. Thanks to Carsten Elsner for his part in this long-distance collaboration.

- PMP participant Travis Cunningham has submitted a pre-print of his new paper, a collaboration with T. J. Christiansen, to Cornell University's arXiv, an open-access repository of preprints.

Their paper is titled: "*Singularities and asymptotic distribution of resonances for Schrödinger operators in one dimension*"

It can be found here:

<https://arxiv.org/pdf/2311.00804.pdf>

Cunningham and Christiansen obtained new results about the high-energy distribution of resonances for the one-dimensional Schrödinger operator. Their primary result is an upper bound on the density of resonances above any logarithmic curve in terms of the singular support of the potential. They also prove results about the distribution of resonances in sectors away from the real axis, and construct a class of potentials producing multiple sequences of resonances along distinct logarithmic curves, explicitly calculating the asymptotic location of these resonances.

ELEMENTARY YARD



Recess Whistleblower,
Stephanie Atherton

Welcome to *The Elementary Yard*, monitored by our “Recess Whistleblower” Stephanie Atherton. Contrary to popular belief (as reinforced by the fresh headshot provided), Stephanie actually graduated elementary school many, many years ago and is currently a junior in college, but she returns to the old stomping ground every Tuesday, where she tutors kids in math, coaches robotics, and occasionally plays tag in the yard. Her favorite things include rainbows, coloring algorithms, and radiating the joy of math for all to delight in.

How to Play in the Elementary Yard

Each iteration, we’ll publish several “elementary” problems for you to warm up with before tackling The Prisoner’s Dilemma, which starts on **Page 21**. We spur you to try them all, and while we will not be taking solution submissions, we do champion problem proposals from readers (to be addressed to stephanie@pmathp.org with the subject line “Recess Stumper”). *Going forward, new stumpers will be presented in each iteration, along with solutions to previous problems.*

THE SURVEYOR’S DILEMMA

This problem was sent to us by Ray Andrews, a PMP volunteer from BC, Canada. He says it is about 11th-grade level.

A mountain has been selected to have a new observatory built on its summit. Because it is very steep and rocky, it would be impractical to build a road up the mountain. Besides, a road would be constantly covered in snow and ice since the mountain is very high and avalanches would be a constant danger. Instead, a tunnel will be dug horizontally from the base of the mountain to directly underneath the summit. From there a vertical elevator shaft will be blasted up to the top of the mountain and all further work will be done via that elevator. To move personnel and tourists to and from the observatory quickly, a cable car will also be installed running between the top and a base facility to be built near the foot of the mountain.

Unfortunately nobody knows how tall the mountain is, so a surveyor is hired to compute that, as well as the length of the tunnel, the height of the elevator shaft and the straight line length of the cable.

The surveyor drives a stake into the dirt, which he calls point ‘B’, where the base facility is to be built.

Using his digital theodolite, which he benchmarks at 1.5 meters above the ground, he measures the angle from point ‘B’ to the top of the mountain to be 45.674 degrees. He then drives directly away from the mountain, exactly 6 kilometers from point ‘B’ over absolutely flat and level ground. He drives a stake at this location and calls it point ‘A’.

Oops! He forgot his theodolite back at base. Never mind, he has an old fractional tape measure and a measuring rod that extends to exactly 20 feet. Estimating the angle to the top of the mountain to be about 30 degrees, he does a quick calculation and then asks his assistant to place the base of the rod 40 feet closer to the mountain than point ‘A’ and hold it exactly vertically. Grabbing a broom stick, the surveyor makes a notch in it 1.5 meters from its end. He then backs away from the rod (towards point ‘A’) until he has a sight line from the top of the rod to the top of the mountain.

Holding the stick exactly vertically, directly in front of



A theodolite is a surveying instrument that measures angles in horizontal and vertical planes.

him and with one end on the ground, he moves slowly backwards until the sightline above just 'grazes' the notch in the stick. (In other words, the top of the mountain, the top of the rod, the notch and the surveyor's eye are all in a perfectly straight line.) He then asks his assistant to measure the distance from the base of the rod to the stick which turns out to be exactly 24 feet.

But the surveyor wants this alignment to take place with the broomstick ending up just over point 'A,' and in fact it is ten feet too close to the mountain. So he asks his assistant to move the rod ten feet closer to point 'A' and they repeat the procedure. However the measurements turn out to be the same. After stopping for a beer with his assistant on the way back to base, the surveyor makes his calculations.



Surveying has been around a very long time, and surveyors eventually triangulated the whole planet.

When they got to mountains and had to do 3D calculations the math got heavy. They had to get to places and endure conditions that would kill most people—and then make these fantastically precise observations and calculations. Put the brains of Isaac Newton into the body of a grizzly bear and you get the idea.

All answers to the nearest meter:

a) How high is the mountain above its base?

The base of the mountain, point 'C', where the tunnel will start, is 3,145 meters closer to the mountain than point 'B'. (Just to be clear, if we take the end of the tunnel as point 'D', then 'A', 'B', 'C', and 'D' are all points on a straight line.)

b) How long will the tunnel need to be?

c) How long will the elevator shaft need to be?

d) What is the straight-line distance that the cable car will have to travel? (Ignore sag, etc.)

Explain why the surveyor redid his measurements. Is his computed height likely to be within one meter? Where is the biggest error likely to be? Are there any issues with his use of imperial units for his measurements at point 'A'?

Fact is, he was a bit nervous about his result, so the next day he went back to point 'A' with his theodolite and measured the angle to the top of the mountain to be 30.847 degrees. What is the new computed height of the mountain?

(The answers can be found on Page 11)

Ray says: Note that all the world's high mountains were measured just like in this problem, long before anyone climbed them. It was 100% trigonometry.

DID YOU KNOW THAT THE WORD "GEOMETRY" LITERALLY MEANS "MEASURING THE EARTH"?

Definition: *Geometry* is the area of mathematics relating to the study of space and the relationships between points, lines, curves, surfaces and the relative position of figures

Etymology: "Geometry" combines two Greek root words: "geo," meaning "earth" and "metron," meaning "a measuring of."



A European and an Arab practicing geometry (painter unknown, 15th century)

INSIDER INSIGHTS

In this column we publish feedback from PMP participants, mentors and volunteers. Do you have something to say that will educate, encourage, entice or entertain your fellow math explorers? Drop us a line at the address on the back page, or if you have access to e-mail, send a message to PMP@pmathp.org. We welcome your input.

The following three essays are excerpted from a prison writing platform that no longer exists. We thought they were worthy of a wider audience. (They have been edited for clarity and brevity.)

On September 23, 2000, I was sentenced to 31 years in prison. I was a very broken-down young man with values that had been extremely damaged and warped by years in gang life. I realized that I had much work to do to find myself in this mess but did not know where to start. With any program I attempted, there would be curriculum I would have to be able to read and understand.

Education has given me a clear understanding of what went wrong in my life. Education has also opened my eyes to how truly lost I had become from the beginning of my childhood forward. Reading has educated me in ways that I cannot fully explain.

For many generations, the narrative about incarcerated people has been that the harder the punishment, the more justice is done in favor of victims and for the community. The hard reality is that victims can become victimizers, and families and communities are destroyed. The other hard reality is that those who have committed crimes are being warehoused in the prison system and forgotten about. Punishment is no longer about justice for victims, but rather it is about profit for those entities that invest in keeping human beings warehoused.

We all know that warehousing human beings does not work, while education does, yet little is being done to educate the public about this reality. Keeping individuals unrefined, incarcerated, and uneducated is a business.

Can you imagine a system where those incarcerated were educated? The culture of prison would truly change. People would make responsible choices, and educated people would hold each other accountable.

We could become people of honor and integrity, who value life dearly and put a great value on the goals, dreams, feelings and the lives of others. Most importantly, we would return to our communities with the understanding that we are capable of contributing to society rather than taking from it and victimizing it. Those in prison need community support and good role models to guide them along the way. I am certain that if communities were inclined to work with incarcerated individuals and understand the profound impact that education can have on each and every one of us, it would not only change the culture of prisons, but it would return trustworthy human beings to families and communities. I believe education is the only way to change the culture of prisons. Education changed my life. I am convinced that it can change the lives of many.

Elmer Cisneros-Alvarado



I made a grave error when I was younger, and I deeply regret it. Now I am determined to show society that people can change. But it's not just about me. It's about providing those who have been incarcerated with the right tools, the right amount of time, therapy, training, and above all, a sense of belonging and trust. With the right support, we can change, rehabilitate, and reintegrate successfully back into society.

There is no use isolating people and then releasing them without the knowledge or skills that are vital in today's world. I have seen and experienced what mental illnesses can do and how it can destroy a life. I was lucky to have taken the necessary steps and received the support I needed. I believe everyone has the capability to change, given the right resources and environment. I have dedicated my life to helping others and giving back to society. I am not the same person I once was, and I am proud of the changes I've made in my life.

Kian Stevenson-Smith



Nineteen years old, barely literate, and despondently ignorant of my aptitude for education, I showed up to prison angry and resistant to positive change. After completing the “mandatory” Basic Education (GED) tests in 2005, I realized that I was a good student and had a thirst for knowledge. I decided from then on that I would take every opportunity for college education that prison could offer me. I went on to earn an Information Technology Certificate, Technical Design Certificate, CNC-Machining Certificate, and an Associate Technical Arts-Business Degree.

Throughout my prison experience I have striven to habitually educate myself as a lifestyle, which has reformed my old criminal thought patterns, behavior, and conduct. Arriving to prison as an emerging adult, I was still very impressionable and open to negative influence from the rest of the prison population.

I made stupid choices and got myself into some trouble: fighting, tattooing, hanging out with prison gang members, and adhering to the “convict code,” which is code for a criminal mindset, used to intimidate/manipulate followers. My determination to educate myself positioned me to mature out of my adolescent developmental deficits, old self-destructive

paths, and created a fertile mental environment for gaining useful knowledge.

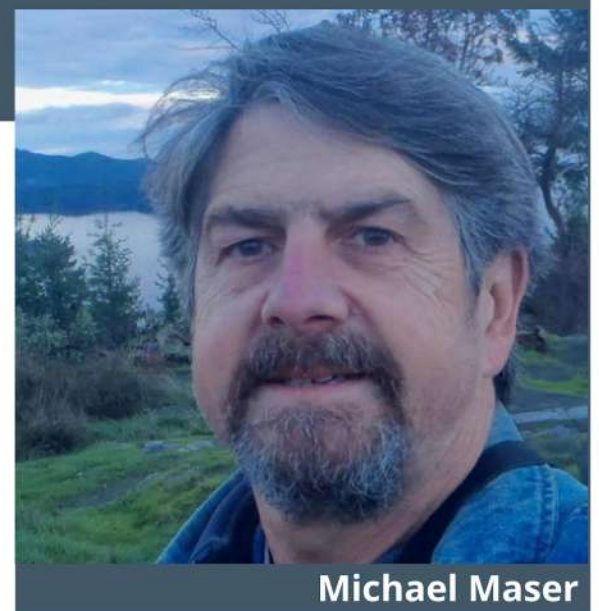
Over the course of my sentence my values have changed. I am no longer intrigued by the unripe schemes of my adolescent self, but instead by personal accomplishments and meaningful contributions to society. I want to gain a college degree so that I can net a higher wage upon release and be a better provider for my wife and family. They are my primary reason and purpose for staying on this path of self-development and education. The great desire to change and engage in self-guided reform must be met by greater opportunities for change. For me, it was the educational opportunities, however limited, that cultivated my success behind bars. I hope to see more opportunities for education in the future.

Randy Brennan 

LEARN YOUR WAY

New to the PMP family in 2023, Michael Maser is an award-winning educator, author and learning coach who teaches Neurobiology and Learning in the Individual Masters program with Antioch University. He received his PhD, focused on learning, from Simon Fraser University, BC, Canada.

His website is www.michaelmaser.net.



Michael Maser

Hello in There! – The Power of Self-Talk

In my previous column I helped identify strategic steps that PMP co-founder Christopher Havens chose to mentally liberate himself from *the Hole* and propel him on a new self-determined (learning) path. These choices helped him overcome serious challenges and achieve goals he never dreamed possible, like becoming a recognized math expert. Christopher has also been instrumental in setting *The Prison Math Project* in motion to help fellow inmates effect positive changes in their lives through engaging with mathematics.

As I see it, through the choices he made and a healthy dose of courage, Christopher created a powerful personal formula guiding his own learning. His remarkable story should give anyone a feel-good

jolt, let alone someone dealing with the day-to-day hurly-burly of incarceration. But – drum roll, please – Christopher’s story also reveals that his achievements are within the reach of what you might also achieve with some additional insights and disciplined focus.

Indeed, it is our human birthright to do remarkable things and we have been doing so throughout our existence. Our personal powers are part of our psychological make-up that aid us – you and me – in overcoming challenges and deep scarring traumas. They help us confront haunting memories and build new foundations for positive personal growth.

Self-Talk: A foundation of personal power

A personal power I wish to highlight in this column is ‘self-talk’ – that is, the things we say to ourselves

that guide us in what we think and believe and thus how we act.

Dr. Lee Pulos, a mentor of mine and a pioneer of self-therapy, changed the world with his insights into this. Lee, who died recently at 95, was a hell of a guy. As a young man he joined the Merchant Marines and sailed around the world. Interested by the varieties of human nature he experienced, he entered medical school where he became a psychiatrist and subsequently explored how humans harnessed the power of their minds to enhance their lives. In the 1970s Lee became the first shrink to help Olympic and professional athletes improve their performance through harnessing their mental powers.

One basic strategy Lee focused on was self-talk – the things everybody, including elite athletes, said to themselves silently and aloud. Lee helped reveal how the things we say to ourselves and the way we say them shape our beliefs and actions. The language we use, he added, consciously and unconsciously encodes the meanings of important experiences and influences us positively and negatively. It makes sense that *linguaging* guides our beliefs and actions. Even the Bible recognizes this: *the Word became flesh*. When I first heard Lee explain this, I found it jarring. I'd never heard anyone acknowledge self-talk though my inner prattle was relentless. It was also often negative: "You can't do that, Michael, you're stupid. You're going to make an ass of yourself if you try it again."

Lee and other counsellors I consulted convinced me my self-talk was normal. I also learned I could adjust it as I wished including changing the language I used and adjusting the tone and volume of my yakkity-yak, like a radio. I did this and it proved to be very helpful. And, full disclosure, I continue to deliberately edit my self-talk each day to help me achieve goals and address challenges I face.

Say that again ...

Think of self-talk as a way of self-coaching, deliberately choosing to talk to yourself in helpful, supportive ways. Make it a habit and you may well create a (free) prescription as potent as any medicine or drug. This brings up another point about self-talk – it triggers ongoing neurobiological events in your body whether you desire this or not. This just happens as part of our human make-up. For example, if your self-talk is very critical or angry your neurological system will likely release certain neuro-chemicals that cause your pulse to increase

and your muscles to tighten. Use softer or supportive self-talk and you may well relax because your body will release different chemicals – all in a split second. Pretty interesting, huh?

I encourage you to experiment with this and find out, or create, your own self-talk formula for personal power based on the guidance below.

1. First, engage your inner learning detective (I've referenced this in previous columns) and notice the qualities of your habitual self-talk: is it critical, loud, aggressive, or soothing? And does this cause you to feel stressed or relaxed?
2. Next, notice that you can change the qualities of your self-talk. In fact, this is totally within your control! And, to repeat, use your own self-talk to *be helpful* – that is, not putting you down or exaggerating details but encouraging you to deal with the facts of a situation and pointing out how you can improve doing, whatever, next time you try. If you aren't experiencing what you desire while engaging in a puzzle or challenge (like a math problem), stop and engage in candid self-talk, and tinker with your own formula until you get the result you desire.

You can even hold a dialogue with your inner voice when it is critical, saying something like, "Your criticism is not helping me right now, so please be quiet." Feeling relaxed or experiencing a little stress is the ideal disposition for learning and your self-talk shapes this.

3. Be realistic in your self-appraisal. That means not being overly critical but also not being so positive or gushy that you overlook the real details of a situation you are focusing on. Use your self-talk to build yourself up, candidly, as a good friend would do. Know the role of your self-talk coach is not to forever praise you or feel sorry for you but to help guide you towards the learning goals truly important to you.

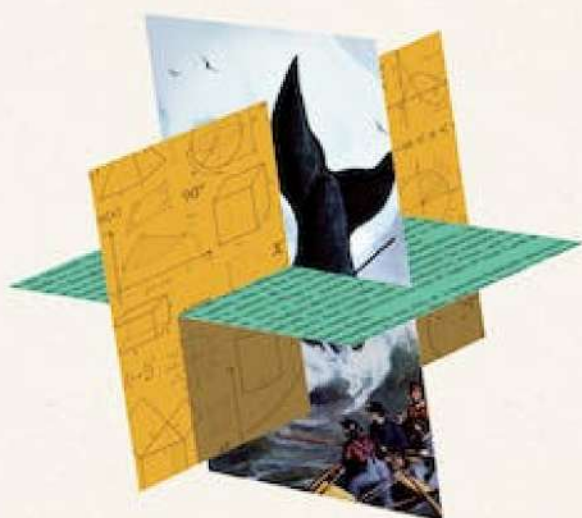
Yakkity-yak, please talk back!

Your homework is to experiment with self-talk, silent and aloud. To notice how you are doing this and shifting what needs shifting as you encounter challenging situations and step up your learning game. If you're encountering learning problems – as you might do if you haven't tackled math problems in a long time, for example – give yourself permission to try and fail and try again. No self-shaming allowed!

In my next column, I'll share some additional strategies to optimize your learning.

BOOK REVIEW CONTEST

Once Upon A Prime



The Wondrous Connections Between
Mathematics And Literature

Sarah Hart

Our reading recommendation for 2024 is *ONCE UPON A PRIME: The Wondrous Connections Between Mathematics and Literature*, by Sarah Hart.

Does this book look familiar? We goofed in our last newsletter when we first featured it, not realizing that it was only available in hardcover and wouldn't be released in paperback until April 9, 2024. Oops! For those of you who have already requested it, you will either have received a hardcover, if allowed in your institution, or no book at all. Therefore, we're featuring it again. If you asked for a copy and didn't get it, our apologies, and please request it again.

If you missed the last Iteration, that's ok! We've got the recommendation from PMP board member Norton Ewart below:

Hello PMPeople,

If I may interrupt your n day (where $n \in \{\text{Fri, Sat, Sun, Mon, Tues, Wed, Thurs}\}$ depending on when you read this, (I'm writing it on $n = \text{Fri}$) I have discovered a book I hope you can get your hands on and read as soon as possible, because each day of delay is doomed to be a less-rewarding day of your life. The book is "Once Upon A Prime: The Wondrous Connections Between Mathematics and Literature" by Sarah Hart.

It has three parts:

- Mathematical Structure, Creativity and Constraint
- Algebraic Allusions: The Narrative Uses of Mathematics
- Mathematics Becomes the Story

Not every chapter will please everyone, but everyone will be pleased by at least one of the ten chapters. I would love to know what our participants think of it.

- Norton Ewart

In 2013 Sarah Hart became one of only five female mathematics professors under the age of 40 in the UK. Her book illustrates how math and literature are far from polar opposites. She connects the dots between the two – and shows how understanding the connection enhances our appreciation of both.

For instance, did you know that George Eliot (the novelist Mary Ann Evans, using a male pen name, which is what a lot of women writers had to do to get published in the 1800s) recommended taking "a dose of mathematics every day"? That Leo Tolstoy used calculus in *War and Peace*? Or that Lewis Carroll sprinkled math puzzles throughout *Alice's Adventures in Wonderland*? (He was a mathematician.) Were you aware that *Moby-Dick* is awash in "lovely juicy mathematics"?

Once Upon a Prime examines the hidden patterns in these and other books, some well known, others more esoteric, revealing layers of beauty and wonder—and teaching a little math along the way.

"By seeing mathematics and literature as complementary parts of the same quest to understand human life and our place in the universe," Hart writes, "we immeasurably enrich both fields." She asserts that poetry is "simply the continuation of mathematics by other means."



Moby-Dick, illustrated by Augustus Burnham Shute, from an 1892 edition of the novel. It's not just the math that's juicy!

Here's how the review contest works:

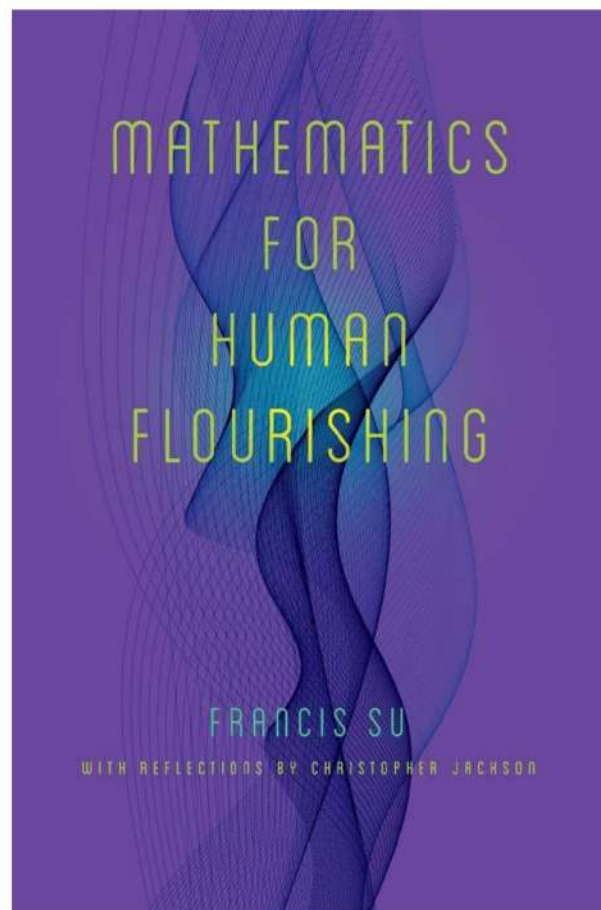
You read the featured book and tell us what you think of it. If your review is deemed by our panel of judges to be the most inspiring to potential readers, it will be published in a future edition of the newsletter and on the PMP website.

If you can't obtain the featured book on your own, please request a free copy from the PMP (address on the back). Once you've read the book, send your review to us via snail mail, or email it to pmp@pmathp.org. There is a catch: we send you the book, you send us a review. Fair? Reviews must be no longer than 1,000 words and may be edited for length or clarity. All submissions for *Once Upon a Prime* must be received by **May 30, 2024**.

BOOK REVIEW REVIEW

No, it's not a typo: we review your book reviews and publish the most inspiring ones.

Our Spring 2023 featured book was *MATHEMATICS FOR HUMAN FLOURISHING*, by Francis Su, a lyrical ode to the connection between mathematics and our common humanity.



Congratulations to Robert Noll for this inspirational review:

I see many commercials each day that claim such-and-such "will change your life!" That was my thought as I approached this book. After all, what product ever actually lives up to the hype?

I committed to dragging myself through it in order to review it and *Wow!* Was I ever wrong!

I started reading on Friday morning and finished it Saturday night. Except for eating and sleeping, everything fell by the wayside. I was drawn in by an amazing writer with an important message.

The writing is clear and concise. It goes into detail without belaboring the topic. At the same time, the reader is treated with dignity; there is no condescension. Rather, it is clear that the writer believes that anyone can understand the message and thus enhance their character.

The premise is that humans flourish when their inherent desires are met. The first chapter lays out the premise and connects it to studying and doing math. Each of the following chapters addresses one of twelve desires, explaining how the meeting of that desire contributes to flourishing.

A desire is introduced and explained. It is then shown how math can contribute to meeting that

desire. At the same time, the virtues that are affected are touched on. In this way the reader can see how a desire leads to strengthening related virtues and thus builds their character. It is apparent that each desire builds on the ones before it. That keeps the whole book cohesive.

There is a subtext throughout, promoting more inclusive and more encouraging teaching. This is related to education in general and math in particular. While I agree completely with that message, I do not think it is the most important message of the book. The most important message is deeper and more subtle.

This is an excellent book encouraging each person to strive toward flourishing. As they do, they won't just change their lives, but also the world around them. Read this book! While you do, I will be enjoying it again to supplement my pages of notes. This time I will go slower, taking more time to explore the games, puzzles, and of course, the math.

Respectfully submitted,

Robert Noll, Pontiac, IL

Congratulations also to our runner-up, Jaroy Gilmer of Fort Dix, NJ, who sent us this review:

I have just finished the book *Mathematics for Human Flourishing* by Francis Su. This is not at all a math book. It is far better. This is the reason that everyone should love math. It explains why I love math, and why GED math students should care. It covers the needs and wants we have as humans and as a collective society. As [a member of an] underprivileged minority, this is now my motivation for continuing to study number theory and to continue to help others, students or not, to continue on the path. I hope to return to society as a productive member, and a good STEM job is the way. This book will change the way you think about *EVERYTHING*. I thank Mr. Su for writing this book and the Prison Mathematics Project for sending it to me. I will keep my heavily annotated copy in my classroom from now on.

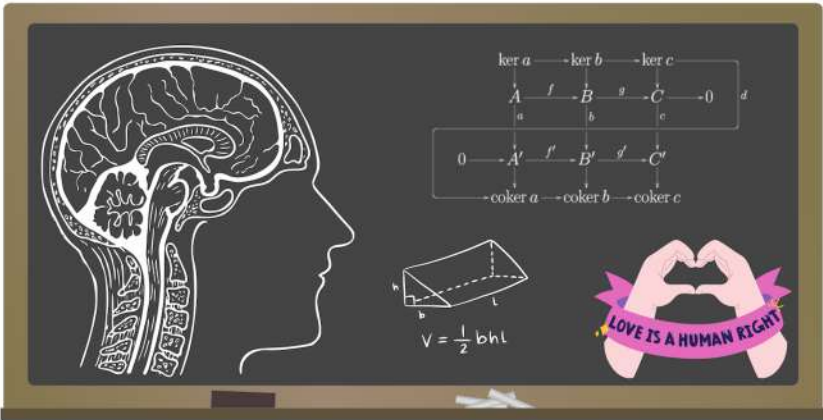
Jaroy Gilmer, FCI Fort Dix, NJ

Answers to the Surveyor's Dilemma on Page 5:

- a) Height: 8,601.5 meters c) Shaft: Same as the height (Easy answer designed to make you suspect a trick!)
- b) Tunnel: 5,255 meters d) Cable distance: 12,022 meters

Close counts! Trig problems don't usually produce 'clean' answers and there will be rounding errors in your calculations. Also, remember that a computed result might be correct 'on paper' but in the real world measuring errors always occur. We've calculated a height of 8,601.5 meters, but what do you think is a realistic margin of error?

As for the new computed height of the mountain at the remeasured 30.847 degrees; using eyeball measurements on a stick is generally likely to be inaccurate. He might be within 0.1% of the original computed height, but that's about an 8 meter variation. As it is, he got lucky and was quite accurate it seems!





Dr. Amy Shell-Gellasch

MATHEMATICS FROM ANOTHER TIME AND PLACE

Dr. Amy Shell-Gellasch is a full time lecturer at Eastern Michigan University. She earned her DA in mathematics from the University of Illinois at Chicago in 2000 and followed that with a post doctorate position at the United States Military Academy at West Point, NY. Her area of research is the History of Mathematics and its uses in teaching. She co-founded and currently chairs the History of Mathematics Special Interest Group of the MAA and is an associate editor of Convergence online journal. Currently she is the Chair of the Michigan Section of the MAA. She conducted research on mathematical devices at the Smithsonian National Museum of American History from 2012-2017.

"Chinese" Stick Multiplication

Multiplication...again. I have warned you, Christopher Havens and I have a thing for multiplication, so I am just going to milk this topic until it runs dry.

This installment of Mathematics from Another Place and Time is a little vague. It is from another place: largely China, Japan, and southeast Asia; and another time in that it has been used in those areas for many years, probably centuries, maybe a millennium. But there is no written record of how long it has been used nor where it started. It is a visual method for the multiplication of two "smallish" numbers (up to say 3 digits each.)

For over 1000 years these cultures have used counting rods on grids to do all four of the basic arithmetic operations. Scholars and bankers carried their own sets of bamboo or ivory counting rods.



Modern copy of a depiction of a Japanese scribe doing a calculation on a grid with rods.

I am going to surmise that this method arose as a shortcut to doing smaller multiplications that did not warrant using a grid (which was often painted on fabric or directly onto a table).



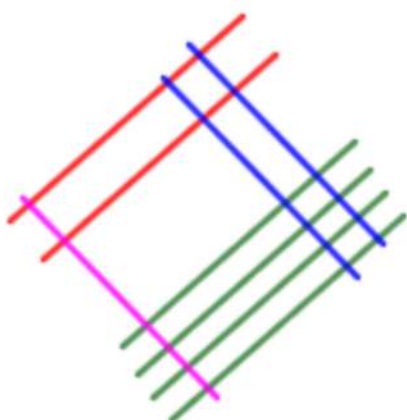
Korean Sangi rods from the 19th century.

[Interesting side note: You pay at a counter, you cook at the kitchen counter, you brush your teeth at the bathroom counter. Why "counter"? Because in medieval Europe, a grid was painted on a high table in establishments and money lenders at which the patron would stand while the proprietor *counted* out coins or made calculations with little counters, called *calculi* in Latin, which means teeth, which leads us to the words calculation, calculus, etc. Cool!]

Children in many Asian countries are still taught this method, and I wish we used it in the US. Having taught adults mathematics for many years, I am always amazed how many adults do not really understand that we have a place-value system, and that that is why our methods for doing addition, subtraction, multiplication, division work...by keeping track of the place values (ones, tens, hundreds, etc.)

So let's lay the rods on the table, so to speak!

Here is a simple example. This is the setup to multiply 12×24 . See if you can figure out how it will give you the product of 288.



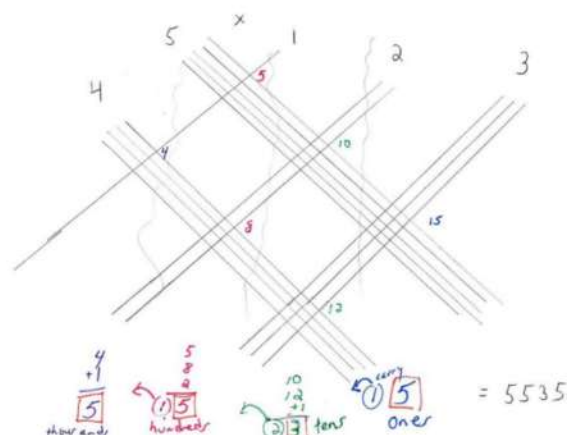
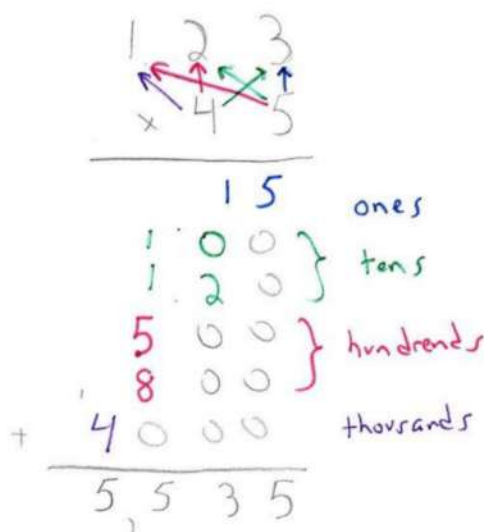
As usual, I'll go find something to do, like pet a bunny, while you work on that.

...
Ok, got it figured out? Right, it is all about keeping track of place value.

The rods slanting up and to the left (slope of -1) represent the number 12, and the rods slanting up and to the right (slope of $+1$) represent the number 24. The number of crossings correspond to the number of units in each place value from right to left: 8 ones, 8 tens, 2 hundreds.

Here is a harder example, with carrying, proceeded by the Western method of doing long multiplication (color coded by place value) so you can see how they correspond

$$123 \times 45$$



As you can see, if you are careful about keeping the rods parallel and roughly perpendicular to the other number's rods, the crossings will line up into columns that represent place value. I have marked those off with squiggly lines. Count all the crossings in a column and you have the total for that place value. For example, the column one from the right represents the two different ways to get "tens": ones \times tens and tens \times ones. Which is the same way we get the tens in the method you learned in school (green in both images). If that total is larger than 9, we carry to the next column.

In short, this method is basically a physical (with rods) or visual (on paper) manifestation of the multiplication you learned. Since all these cultures use the same base-ten place-value system, all the arithmetical operations are the same. What differs is how we present them. This method has the advantage of being easy to learn and easy to remember. None of those hard-to-remember rules of the Western methods.

Voila!

Have a little time on your hands? Calculate 56789×98765 using the stick method.

If you would like to suggest a topic or submit a piece for That is So Cool! or Math from Another Time and Place, please email me at amy@pmathp.org

In honor of Pi Day (March 14), here are the first 10,000 digits of pi!
We trust this is enough to keep you memorizing for quite a while ☺

$\pi \approx 3.14159\ 26535\ 89793\ 23846\ 26433\ 83279\ 50288\ 41971\ 69399\ 37510\ 58209\ 74944\ 59230\ 78164\ 06286\ 20899\ 86280\ 34825\ 34211\ 70679\ 82148\ 08651\ 32823\ 06647\ 09384\ 46095\ 50582\ 23172\ 53594\ 08128\ 48111\ 74502\ 84102\ 70193\ 85211\ 05559\ 64462\ 29489\ 54930\ 38196\ 44288\ 10975\ 66593\ 34461\ 28475\ 64823\ 37867\ 83165\ 27120\ 19091\ 45648\ 56692\ 34603\ 48610\ 45432\ 66482\ 13393\ 60726\ 02491\ 41273\ 72458\ 70066\ 06315\ 58817\ 48815\ 20920\ 96282\ 92540\ 91715\ 36436\ 78925\ 90360\ 01133\ 05305\ 48820\ 46652\ 13841\ 46951\ 94151\ 16094\ 33057\ 27036\ 57595\ 91953\ 09218\ 61173\ 81932\ 61179\ 31051\ 18548\ 07446\ 23799\ 62749\ 56735\ 18857\ 52724\ 89122\ 79381\ 83011\ 94912\ 98336\ 73362\ 44065\ 66430\ 86021\ 39494\ 63952\ 24737\ 19070\ 21798\ 60943\ 70277\ 05392\ 17176\ 29317\ 67523\ 84674\ 81846\ 76694\ 05132\ 00056\ 81271\ 45263\ 56082\ 77857\ 71342\ 75778\ 96091\ 73637\ 17872\ 14684\ 40901\ 22495\ 34301\ 46549\ 58537\ 10507\ 92279\ 68925\ 89235\ 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Paul Sambursky

PARTICIPANT SPOTLIGHT

by Claire Finlayson

“

Well, someone has to keep these math geeks in line and jump on every little spelling mistake...

I'm a writer from BC, Canada, so don't try to bust me for using British spelling, like "cheque" and "colour," okay?

I am a PMP superfan, and my rudimentary math skills have not proven to be an impediment to my participation in this wonderful organization. It's the unique, true-life stories of people that interest me. So I write profiles of participants, volunteer mentors and others involved in the program.

”

If there's someone you'd like me to spotlight, here's how you can contact me:



www.clairefinlayson.com



claire@pmathp.org



Prison Mathematics Project,
10810 N. Tatum Blvd Ste 102-998
Phoenix, AZ 85028



Claire Finlayson

Interview with Paul Sambursky, North Dakota Department of Corrections

CF: Hi, Paul. Thank you so much for agreeing to this interview. The executive director of the PMP, Ben Jeffers, suggested I contact you, as apparently you're hot stuff in the PMP program and you teach math in your facility. Ben didn't call you hot stuff, by the way, I did—but I mean that in purely mathematical terms, of course.

PS: Of course!

CF: Could you tell our readers a little about yourself?

PS: I am an inmate at the North Dakota State Penitentiary. This is our state's maximum security

facility. I am 49 years old and have been here for 20 years now. I have just under four to go. I had just left active duty with the US Marine Corps after completing my tour and receiving an honorable discharge. But I was one of those guys who did not transition well, and quickly earned myself a 30 year sentence.

CF: I have heard similar stories from a number of inmates who have returned from military tours, many with PTSD and/or addictions, only to end up in prison, their lives and the lives of their loved ones shattered. It seems that soldiers are over-represented in US prisons. So, you received a thirty-year sentence in the prime of your life ... How did you cope?

PS: I didn't. In fact I had great difficulty transitioning to this life.

CF: I'm interested in hearing more about that, but first, may I ask, do you have a family?

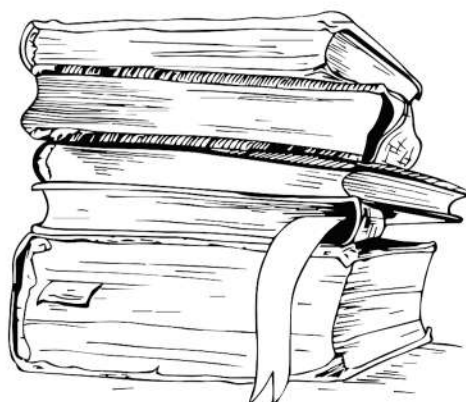
PS: I once had a very beautiful wife and I have four amazing children. Sadly, I did lose them. Twenty years of incarceration will strain and destroy pretty much any relationship, especially when you are incarcerated hundreds of miles away from your children.

CF: I imagine it would be a challenge for anyone in your predicament to sustain relationships, but the distance would be the final kicker. And yet despite the loss of your family, I understand you've been able to haul yourself up by the bootstraps and create a meaningful life. How did you do it?

PS: When I got to prison I was bitter and angry, and I soon earned myself a very lengthy stay in solitary confinement. It was there that I finally hit rock bottom and decided that if I was going to live in this place—and at that point I had not yet finalized the decision to live at all—I was going to have to do better and be better. So I decided to tap into the discipline I had learned in the Marine Corps to improve my life.

CF: So you decided to swim rather than sink. What strategies did you employ?

PS: I started voraciously consuming textbooks every day instead of watching TV or reading novels. I wanted to learn more about science and physics, but I had no math ability. I had always thought I was learning-disabled in math, so I had never tried before. But math seemed like the gateway to things I wanted to know, so I *had* to learn.

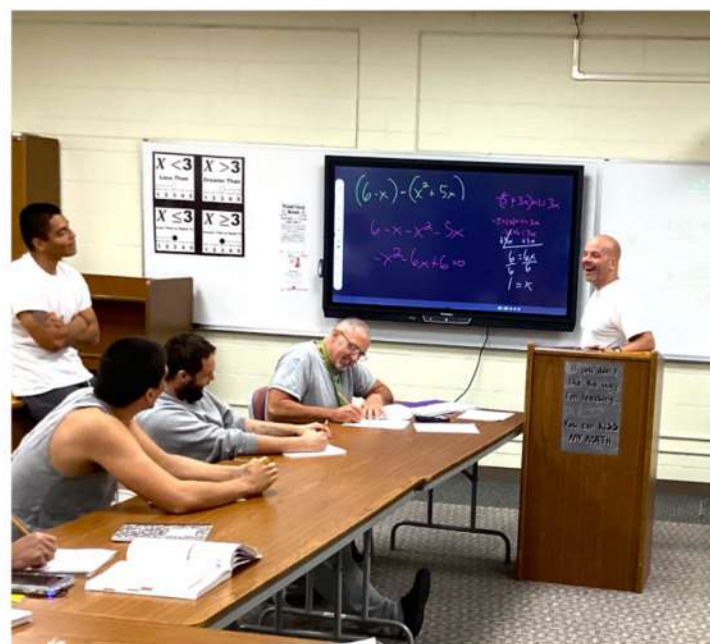


When I started I could not even add basic fractions. I asked Nathan, our librarian, for introductory math books. Slowly I started mastering the fundamentals. Once I started having some success, I decided to do every single problem in the book. I set aside 90 minutes per day, every day, to learn math. I quickly completed the book and asked for more. And more. And more... Whenever I got stuck I would ask another inmate or just figure it out myself. Soon, I had consumed all the math books in our library, and our librarian went out of his way to find me even more.

CF: It's like you were thirsty, and this librarian brought you water. I wish every prison library had a Nathan! Anyway, so you went from thinking you were "learning-disabled" to devouring every math book you could get your hands on.

PS: By then I knew that math, and the discipline of math, was what had saved my life and given me hope for my future, so I wanted it to be a big part of my life going forward. But I could not find any math programs anywhere in the country that would accept prison inmates.

CF: That's something the PMP is well aware of and working to address, of course. But it sounds like you weren't going to let anything stop you. And I understand you've now made the transition from math learner to math teacher?



Paul Sambursky teaching a math class

PS: That's right. Seven years ago I got hired as a GED/ABE tutor. It was a good job, about 40 hours per week, and it kept me out of trouble. But not only that: I found out that I absolutely love teaching, and helping others. This was a good thing for the inmate population here as well, as we have over 100 GED students out of a population of 800-900 men. I didn't even have a college education. I wanted one, but had no money or ability to get one. So I kept up my own personal study the "Good Will Hunting" way. I studied more advanced mathematics, physics, finance, and every college text book I could find. I still kept that 90 minutes a night sitting on the floor in my cell studying.

CF: So in effect you became your own college instructor. That's remarkable, Paul.

PS: At some point I started helping the staff math teacher whenever she got stuck, and eventually the principal allowed me to substitute for her when she was unable to make it to work. This got me enough attention that when I reached out to my church and to everybody I know, I was able to find a wonderful group of benefactors who offered to pay my tuition. It took a number of years, but I have just graduated with my Bachelors degree in Business Administration and Management from Adams State University.



CF: Congratulations to you and your benefactors. Looks like their investment in you is paying off. So while earning your degree you continued tutoring and working as a substitute teacher?

PS: Yes, but then about two years ago the principal asked if I would just become the full time math instructor here. I love it! And the inmate population loves that one of their own is their teacher. I ended up with four or five classes—about 40 adult students per day. I work very hard to simplify the process and teach each person at the level they are at.

I love bringing to my students my joy and appreciation for math and what it did to change my life around. My speciality is imparting to them just how important math is, and how it can be applied to their daily lives. I graduated three math students today and Claire, it's a wonderful feeling. That is how I count my wins. Most of these guys are like I was: they can't even do long division when I get them. They dropped out of high school or failed, or for some other reason were just not that good or interested in school when they were younger. But when they graduate, I see men who are that much less likely to come back to prison, men who get a chance to be better fathers, men who can earn a decent living ... thus being better able to provide for their families. These are the wins I am truly grateful for and why I work so hard at what I do.

CF: I doubt that these graduating students will be coming back to prison EVER! That's a win for them, for you, and for society at large.

Most of us can remember a special teacher in one subject or another who lit a fire in us. I wonder, down the road, how many of your students will be naming you as that teacher who helped alter their destiny ...

So you're still teaching GED/ABE to every math student in your institution?

PS: Yes, but there have been changes to the curriculum. In order to get a GED now, students have to have a good understanding of Algebra 1 and Basic Geometry, as well as a general understanding of many Algebra 2 topics. Last year, Ashland University started offering a variety of college classes in this prison. They quickly found out that many of their students do not have the math background to get higher education degrees. My principal asked me to fill in as an instructor for these students as well. So I now have quite the one-room school house of math. In any given hour, I can teach basic math, basic algebra, advanced algebra, geometry, developmental college math, advanced finite mathematics and pre-calculus.

CF: That's more than a one-room school house, Paul! It sounds more like a three-ring circus, with you as trapeze artist, lion-tamer and juggler—oh, and ringmaster. It would definitely keep you on your toes.

PS: Yes, it does. My classes are full, fun, and the students really are a joy to teach. I have a poster on my podium that says, "If you don't like the way I'm teaching, you can kiss my MATH!" It always gets a chuckle out of the new dudes. Oh, and by the way, whoever came up with PMP flyers that can get torn out of your newsletter and placed up (with proper permission) was a genius. I love it, and I will definitely be doing that.

CF: Excellent. We hope other incarcerated readers do the same. The genius was Walker Blackwell, by the way, who is one of the PMP's founding fathers, though he is still a high school student.

PS: This year, my principal offered me yet another new teaching position as the Personal Finance and Financial Literacy instructor for the entire penitentiary. So, I still have three full math classes every day, but now one of them has been turned into a finance class for those in general population who just want to know more or are getting ready to go back to society.

CF: I want to say something about your institution here. The staff have obviously recognized your potential and have kept giving you new opportunities. In fact, I reached out to your principal, Rebecca Deierling, and she told me that from the time you began working as a tutor seven years ago, it was obvious that you had a real talent for working with others. In fact, allow me to quote from her e-mail:

"Paul has a way of explaining math to students that helps them understand it. He will attack the problem in many ways until he finds the path that his student can understand. When our math teacher was out sick or on vacation Paul was quick to jump in and teach those classes. I immediately noticed that he had missed his true calling as a math educator. When our math teacher

left for different opportunities, I approached Paul and asked if he would be willing to teach the class until we could find a replacement. He was so excited for the opportunity. Something quickly started happening. Our students were starting to pass their Math Official GED tests in a quicker fashion. We stopped looking for a replacement teacher and continued to have Paul teach the math classes.

This has been an amazing opportunity for all of us. Not only do we have a marvelous math instructor, but our students have been able to see one of their own step up in a leadership role and make real change within our institution. We are truly blessed to have Paul as part of our team!"

I can say from my own experience with the PMP that not all prisons are that forward-thinking.

PS: This place has seen many changes over the years. One of the most important is that we have become a national leader in prison reform. We have tried to adopt the "Norway Model" and have become very progressive in how prisoners are treated. My job is just one example of that.

CF: Do you wear uniforms or civilian clothes?

PS: We do have state provided clothing, much like a uniform, but we have the ability to purchase jeans, grey t-shirts, and grey sweatpants/shirts. So, we get pretty clever when it comes to our wardrobe selections. Because my job pays a little bit better, I am able to purchase jeans, and that is what I typically wear.



CF: As we see in the pictures your warden kindly sent us! Very snappy. But kidding aside, that small concession must go a long way toward making prisoners feel more like individuals and less like ants in a glass terrarium.

I'm not so much about the math (as I confess openly in my byline) as I am about the people doing the math. Your story starts with you as an angry, lethally depressed and dangerous criminal who couldn't add fractions, who metamorphosed into a thriving human being and gifted math educator who is an asset both to the staff and the inmates in your institution! *Blessed*: that's the word your principal used. I don't imagine it's a word that state penitentiary officials throw around lightly when speaking about inmates.

PS: Yeah, well, Rebecca is pretty much the main reason why I am allowed to be a full-time teacher as a prison inmate. She's really committed to rehabilitation versus vengeance and punishment. She is a wonderful person.

CF: It sure sounds like it.

While this is all quite incredible, I'd like to bring the conversation around to your association with the PMP. How did you hear about us?

PS: About 6 months ago I found a podcast called "Math Therapy" and had one of my staff members print out a nice little homemade flyer to advertise it. I was given permission to put it up in my classroom and in the hall. Then I found out that the purple-haired woman who runs the podcast is also associated with PMP.

CF: You're talking about our very own Vanessa Vakharia, the "Lady Gaga of Mathematics," who writes a column for this very newsletter! Her hair is only purple sometimes, by the way. She's a blast, but she's very serious about helping people get over their math trauma. You and Vanessa would have a lot to talk about.

PS: Then one day, one of the staff members here learned about PMP online and printed off a copy of the newsletter. I quickly contacted PMP and asked for a mentor who was a math teacher so

that she/he could give me some tips, as I was self-taught. They put me in contact with Luke, a college math instructor who gave me a lot of encouragement and advice.

CF: PMP has mentors waiting to assist any inmate who has made a commitment to rehabilitation, no matter their level of math maturity, but in your case that's an interesting niche. I'm glad we were able to pair you with someone who fit the bill.

PS: Claire, once I learned the true beauty and necessity of math, it helped to not only turn my life around but has opened doors for me. I had no hope and was getting ready to give up on life, but through the discipline of learning math, I have found such beauty and hope for the future.

CF: Your words are an echo of those spoken by PMP founder Christopher Havens, who underwent a metamorphosis like yours during his own journey into mathematics. Thank you for your candor in sharing your story, Paul. I have found this conversation to be most uplifting.

PS: Well, thank you for this opportunity.



*From left to right, the graduating students are:
Robert, Joe, Danny, Noah, and Dakota.*

Two of these students are now out in the "real world."

THE PRISONER'S DILEMMA

From the Problem Warden

I am delighted to be part of the Prison Mathematics Project through The Prisoner's Dilemma, as its (honorary) "Problem Warden." My love of mathematics – and especially sharing the joy it can bring – have been a part of almost everything I've done as a professor, financial analyst, parent, founder of the National Museum of Mathematics, author of the Studio Infinity blog, and the most recent ex-editor of The Playground (the problem column of Math Horizons, a magazine chronicling the world of math for the Mathematical Association of America). I look forward to seeing all of the new methods and creative questions that you come up with as we face the many dilemmas to come, together. And don't worry, as a one-time specialist in mathematical logic, this Warden will keep a sharp eye out for any infractions in your reasoning!

– Glen Whitney



Glen Whitney, AKA
The Problem Warden

This time, you'll find solutions to problems from two iterations ago. There will only be two iterations of the PMP newsletter for 2024, so you'll have plenty of time to work out some creative solutions! Even with that extra time, we know some problems are hard nuts to crack, but it's worth persevering. So we're also including **D13** "The Experbola" again for another round, since the Problem Warden has only received a single proposed solution. In case you tried it earlier and couldn't get a foothold, this time we've added a hint or two, so look it up in the *Second Hearing* section. And of course, we have three new problems for you to dive into, as always. Happy solving!

This problem first ran in Iteration 7, but is repeated here because the defining relationship for a sign-flipping function was originally misprinted. The Prisoner's Dilemma will accept submissions using either definition; if you've already solved it with the earlier equation, you may enjoy trying to solve this corrected problem.

D14: Flip Functions

Contributed by Yagub Aliyev, ADA University, Azerbaijan

Call a function f mapping the real numbers to themselves a sign-flipping function if for all $x, y \in \mathbb{R}$,

$$(x - y)[f(x) + f(y)] = (x + y)f(x - y)$$

For example, the identity function $i(x) = x$ is a sign-flipping function because $(x - y)(x + y) = (x + y)(x - y)$ by commutativity of multiplication. Are there any others? Find all sign-flipping functions.

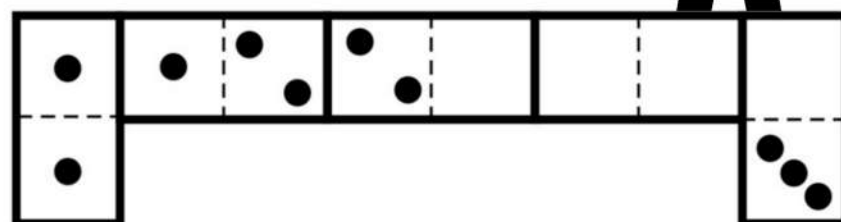
D18: Domino Design

Contributed by Ian Stewart, University of Warwick

Since we have solutions to a previous problem from Ian below, it's time for another one of the offerings from Warwick's Duke of Dilemmas, this one based on familiar playthings. **Figure 1** shows an arrangement of five dominoes such that adjacent squares of two different dominoes contain the same number of spots. But more interestingly, the two dominoes at the end total five spots, and so do the three in the middle. There are three other ways to achieve this, not counting left-right reflections as different. Can you find them?



Figure 1: A row of dominoes with five pips at the ends and five in the middle.





D19: Covid Conundrum

Contributed by Arsalan Wares, Valdosta State University

Well, if Ian has a new problem this time, then Arsalan deserves one, too. Showing his versatility, this time he's proposed a Dilemma that is not based on one of his beautiful diagrams. Instead, suppose you stood on line as part of a group of 30 people waiting to receive a vaccine. After everyone finished, you were told that ten members of the group turned out to have cases of Covid. You never came down with the infection, so you couldn't have been one of the ten. If you were neither first nor last in line, what is the probability that at least one of the two people next to you in line had the virus (but you got lucky and didn't catch it anyway)?

D20: Geometry Game

Contributed by Dave Richeson, Dickinson College

Of course, we wouldn't want to go a whole issue without a new geometry puzzle, so fortunately first-time proposer Dave has us covered. You and a buddy start with a rectangular cake, represented by rectangle $ABCD$ in **Figure 2**. Then you get to pick any point E between C and D , and the cake is sliced with two straight cuts, from B to E and E to A . Then it's your friend's turn to pick a point F between B and C , so that the cake will be sliced from D to F and F to A . The four straight slices intersect at points G , H , and I as shown in **Figure 2**. You receive piece $AGHI$ of the cake, and the other player receives pieces DEG , $CEHF$, and BFI .

Determine all points E you can choose on side CD that will prevent your buddy from ending up with more cake (in all) than you do.

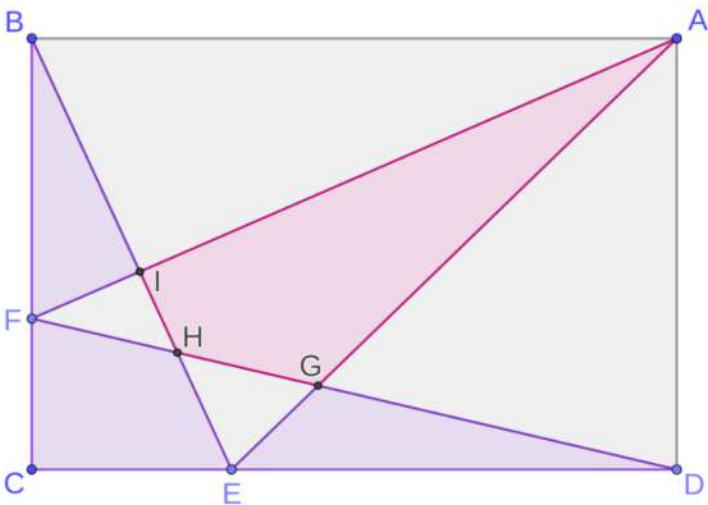


Figure 2: Game of Slices; you get the pink, they get the purple.

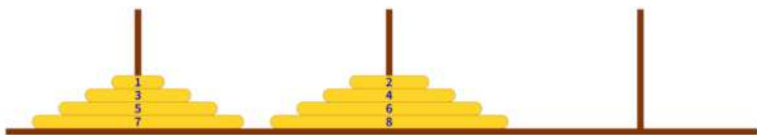


Figure 3: The starting position for Half-and-half Hanoi.

D10: Half-and-half Hanoi

Andreas Hinz, LMU München

You may be familiar with the Tower of Hanoi puzzle in which you start with eight discs stacked on one of three pegs, each disc smaller than the one below it. The other two pegs are empty. The goal of the puzzle is to move the entire stack of discs to a different peg, according to the following rules:

- (a) Only one disc may be moved at a time, from the top position on one peg to the top position of a different peg.

- (b) No disc may ever be placed on top of a smaller one.

For convenience, we number the discs from 1 to 8 in order from smallest to largest. In this variation, you start with discs 1, 3, 5, and 7 on the first peg, and discs 2, 4, 6, and 8 on the second peg, as shown in **Figure 3**. Your goal is to end up with all eight discs in order on the third peg, following both rules (a) and (b). What's the fewest number of moves needed to accomplish this goal, and how do you do it? How do you know your solution is minimal?

SECOND HEARING

In this portion of the column, we revisit prior dilemmas to share solutions that readers have come up with, or report on their status.

The *Prisoner's Dilemma* received solutions from PMP participants William Keehn and Robert Noll. Both writeups were excellent; we follow William's presentation here. He begins by showing that the minimum number of moves to solve the usual Tower of Hanoi puzzle with n discs is $H_n = 2^n - 1$, by breaking the problem into shifting the top $n - 1$ discs (using H_{n-1} moves), followed by moving the n th disc, followed by shifting the $n - 1$ smaller discs back on top of it (using another H_{n-1} moves). He then solves the resulting recurrence relation $H_n = 2H_{n-1} + 1$.

Then considering the problem Prof. Hinz poses, William notes that every move is reversible, so if you were to play a film of the optimal solution backward, it would necessarily be the optimal way to turn a pile of eight discs into the configuration shown in **Figure 3**. But to do this, you must first use H_7 moves to shift the top discs, one move of disc 8, H_5 moves to uncover disc 6, one move to put 6 on top of 8, H_4 moves to uncover disc 5 and one more to put it on 7, H_2 moves to uncover disc 3 and one more to put it on disc 5, and two final moves to put disc 1 on 3 and disc 2 on 4. (This sequence of events is beautifully depicted in William's drawing shown in **Figure 4**.) Using the formula for H_n , these phases total 182 moves, the minimum possible.

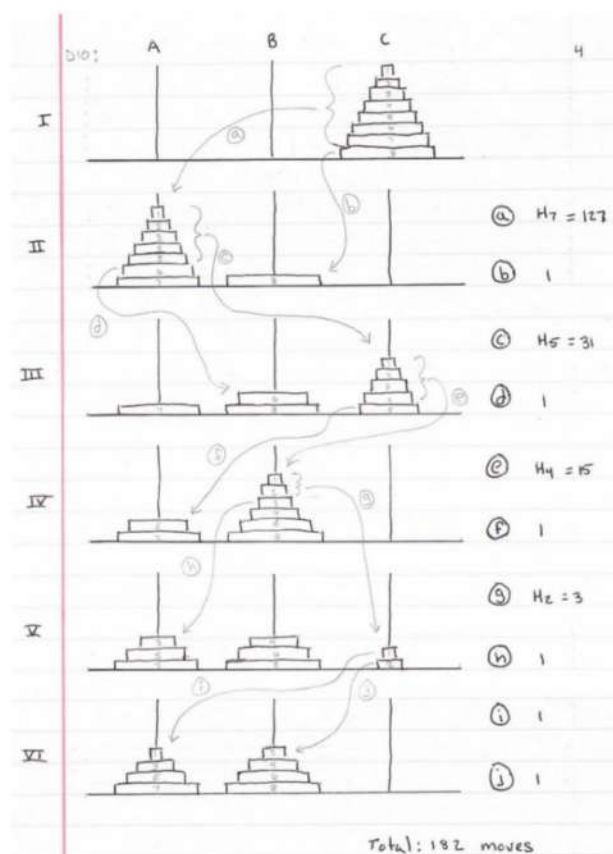


Figure 4: Separating Hanoi discs by parity.

D11: Octosection

Arsalan Wares, Valdosta State University

For this setup, refer to **Figure 5**. Square $ABCD$ has been dissected into eight equal-area rectangles, as shown. The width of the rectangle shaded red in **Figure 5** is 35 units. What is the area of square $ABCD$? For a bonus challenge, can you devise and solve a similar problem that dissects a square into eleven pieces? For this one, we heard from

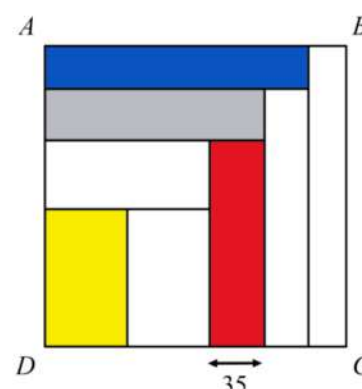


Figure 5: A square dissected into eight equal-area rectangles.

William and Robert again, but also from first-time submitter Blazej Kot, a PMP participant. All solutions proceeded roughly as so: Let $s = |BC|$ be the length of one side of the square. Since the rightmost white rectangle has area $s^2/8$, its width is $s/8$, and so the length of the blue rectangle is $7s/8$. Hence the width of the blue rectangle is $s/7$. That makes the height of the white rectangle below it $6s/7$, corresponding to a width of $7s/48$ (since again its area is $s^2/8$). Subtracting the widths of the two white rectangles on the right shows the length of the grey rectangle is $35s/48$, corresponding to a width of $6s/35$. Subtracting the widths of the blue and gray rectangles shows the height of the red rectangle is $24s/35$, yielding a width of $35s/192$. But we are given that the width of the red rectangle is 35, so we know $s = 192$ and hence the area of the outer square is $192^2 = 36,864$.

The bonus challenge catalyzed a burst of creativity: Every submitter proposed at least one 11-piece variant, some devising multiple possibilities. And every proposed "undecasection" was different! Sadly, we can only summarize this glorious profusion with the gallery of diagrams presented in **Figure 6**.

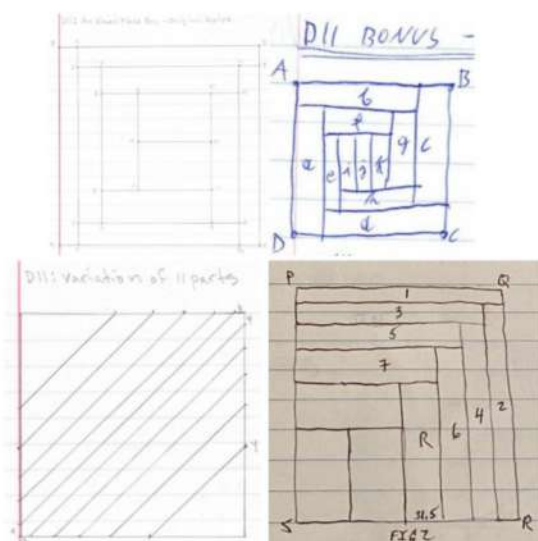


Figure 6: So many problems with dividing a square into eleven pieces!

D12: Strange Coincidence

Ian Stewart, University of Warwick

This Dilemma was about a family divided by math, so to speak. We listened in on their conversation:

–“That’s amazing!” said Mathophila. “The squares of three consecutive positive whole numbers add up to the same number as the squares of the next two numbers.”

–“Oh, no big deal, that must happen all the time,” replied her brother, Innumeratus.

–“I don’t know, it seems like an amazing coincidence to me!” retorted Mathophila.

Find the smallest collection of numbers with the described property, and settle the argument between the siblings: determine whether there are infinitely many sets of numbers as described by Mathophila, or a finite number (and if so, figure out exactly how many solutions there are).

In a lovely sort of “cumulative submission series,” all three solvers of **D11** also cracked this one, and they were joined by PMP participant Chris Bistrski. All of the solutions proceeded algebraically by assigning a variable to the value of one of Mathophila’s numbers, which determines the others since her five numbers are all consecutive. But Robert pointed out that the algebra works out most smoothly if you let your variable n represent the *middle* number. Then you get the following equation

satisfied by Mathophila’s numbers:

$$(n-2)^2 + (n-1)^2 + n^2 = (n+1)^2 + (n+2)^2.$$

This relationship simplifies to $n^2 - 12n = 0$, or factoring, $n(n-12) = 0$. Hence n is either 0 or 12, but the smaller value would make some of the numbers negative. Hence the only set of five numbers with Mathophila’s properties is $10^2 + 11^2 + 12^2 = 13^2 + 14^2$. Amusingly enough, the common value of the left and right hand sides is the number of days in a (non-leap) year.

Chris, however, took this conclusion as merely the start of an in-depth exploration. He first asked what would happen if Mathophila had a set of $2r+1$ consecutive numbers so that the sum of the squares of the first $r+1$ of them is the sum of the squares of the next r of them (so that in Mathophila’s original case, $r=2$). He proved in this more general setting that the middle number of the set must be $2r(r+1)$, or just four times the r th triangular number. For example when $r=3$,

$$21^2 + 22^2 + 23^2 + 24^2 = 25^2 + 26^2 + 27^2.$$

See if you can match Chris’s accomplishment (you might want to try using induction on the value of r , or you may find another method). Not content to stop there, Chris asks whether for any whole numbers r and k both greater than one, the sum of $(r+k)$ consecutive squares is equal to the sum of the next r consecutive squares. For example, the simplest case of this question is $r=k=2$, but the equation corresponding to the first one above simplifies to $2n^2 - 14n - 7 = 0$. This quadratic has only one positive solution $n = (7 + 3\sqrt{7})/2$, so there are no whole-number solutions. In fact, Chris was able to show that there are no whole-number solutions for *any* r when $k=2$. But that still leaves a whole lot (infinitely many!) of other possibilities for k , so if any reader is able to find other k value(s) that allow whole-number solutions (for one or more values of r), we’ll be happy to print these findings here in the *Prisoner’s Dilemma*.

D13: The Experbola

William Keehn, PMP Participant

Consider all of the points where $x^y = y^x$ such that x and y are distinct positive real numbers. You get a curve similar to a hyperbola, that William has dubbed the “experbola.” **Figure 7** shows the hyperbola and the experbola side-by-side.

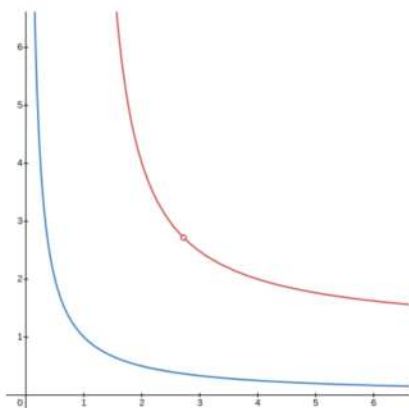


Figure 1: A hyperbola (in blue) and the experbola (in red). Courtesy of desmos.com.

And William is brimming with interesting questions about the experbola, so **Dilemma 13** has lots of parts (feel free to submit answers for any or all of them):

- Find all of the points on the experbola where both coordinates are whole numbers. (And explain why there aren't any others.)
- An asymptote of a curve is a straight line that the curve becomes arbitrarily close to but never touches. For example, the hyperbola in **Figure 7** has two asymptotes: the lines $x = 0$ and $y = 0$. Does the experbola have any asymptotes? If so, find them.
- The condition that x and y are distinct is needed because whenever they are the same, the equation $x^y = y^x$ is trivially satisfied. So without that condition, the experbola would also include the entire diagonal line $x = y$. But as you can see in **Figure 7**, that condition also cuts the experbola into two disconnected pieces. However, there is a unique single point on the line $x = y$ that you could add back to the experbola to “complete” it and make it a single, continuous curve. (That's the point indicated by the small red open circle in **Figure 7**.) What are the coordinates of that point?
- Find (non-constant) functions $f(t)$ and $g(t)$ such that the point $(f(t), g(t))$ is on the ex-

perbola for all positive real numbers t . For this part, William offers a hint by way of an example: $(\sqrt{3})^{3\sqrt{3}} = (3\sqrt{3})^{\sqrt{3}}$. Such a pair of functions is called a *parameterization* of the experbola, or at least of the part of the experbola it lands on.

Since we only received one solution to **D13**, just before press time, we're going to give you one more iteration to try it. And here are hints for (a) through (c) as well to help you get started. For (a), remarkably the easiest way to solve this question about integers is to use some well-known properties of the transcendental function $f(t) = e^t$, namely that it is concave up and its slope at $t = 0$ is 1, giving you the inequality $1 + t < e^t$ for all $t > 0$. You can apply this fact to the problem at hand by assuming by symmetry that $y > x$ and writing $y = x + z$ for some positive integer z . If you're still finding this challenging, don't be too alarmed: this part of the problem appeared on the 1960 Putnam Competition, widely regarded as the most difficult collection of math challenges to be posed to college students each year.

For (b), just ask yourself whether it could happen that $x < 1$ while $y > 1$. And finally for (c), even though it seems like a very different question, you can begin in almost the same way as for part (a), just dropping the requirement that x , y , and z be integers. Or you could solve part (d) first and then try to see if there is any t for which $f(t) = g(t)$.

Submission Guidelines

Solutions to problems published in The Prisoner's Dilemma, and proposals for new Dilemmas, are welcome. For solutions, please clearly indicate the Dilemma number being solved. If a problem has multiple parts, you may submit solutions to any individual part or parts. Solutions to the Dilemmas in this newsletter must be received by the deadline of **May 15, 2024**, and they will appear in the iteration after the next one. Dilemma proposals will be considered on an ongoing basis. All submissions should be addressed to Glen Whitney/Prisoner's Dilemma either by email at dilemma@pmathp.org (in which case PDF format is preferred, if possible, although any reasonable format will be accepted), or by mail at:

Prison Mathematics Project
Glen Whitney/Prisoner's Dilemma
10810 N. Tatum Blvd. Ste 102-998
Phoenix, AZ 850

CLOSING COMMENTS

Ladies, gentlemen, and folks of all expressions,

Let me begin by saying that this issue of the newsletter is something special. For us here at the PMP, these words are coming from our hearts, and as I write this message in the final hour of 2023, the promise of the new year is upon us. By the time you read this it will be spring and we are fast approaching our mathematical holy day of Pi Day on March 14. As I speak to you in this state of "in-between," there is a feeling within me identical to the feeling that led to the creation of the Prison Mathematics Project.

The PMP was launched as a way to introduce prisoners to the concept that communities exist around some of the things that we love most. For us here, it's mathematics. When we become engaged in a mathematical practice and immersed in the community and the culture that surrounds it, the most profound sort of rehabilitation can occur. I know, because I am experiencing it as I live my dreams inside a prison. I am determined to realize a version of myself that will be able to look back and know I exceeded the expectations I set with this new year's hopes and dreams.



It has always been my vision that this organization should become a beacon for incarcerated mathematical minds, regardless of their experience, so that they can **feel** just a tiny bit of what is overflowing from my own heart through this grand endeavor of numbers. It has also been my dream that someday the PMP would diverge from the organization that Jack Smith, Walker Blackwell and I built so many years ago. What I mean is that I want to see PMP grow from my own idea into the ideas of the next generation who will lead our organization into the future. And I am seeing this change happening.



Christopher Havens

It's a humbling experience to share my ideal math community with all of you, and it's also humbling to feel how our new Executive Director, Ben Jeffers, has inspired this beautiful divergent growth that I have long yearned for. I see much progress. I see teamwork and I see differences that inspire innovation.

So let us now make a new goal together, that on this Pi Day of 2024, we endeavor to understand the mistakes of our past, whether we are incarcerated or not. Let us set aside a single one of our vices and apply that new opportunity of time to one of our mathematical goals. Let us engage in a community that exists around that **thing** in our lives that we love the most. By doing this, we have found home, regardless of any physical location.

To our readers, this newsletter's purpose is to share mathematical culture and impart that feeling of community. As a co-founder of this shindig, I am calling to all of you, whether free people or incarcerated scholars, to use this newsletter as a platform for sharing the story of your own mathematical journey. This is my Pi Day wish, so that we can all enjoy our differences and shared humanity through the lens of mathematics.

Until next time,
Christopher Robin Havens



PMP
PRISON MATHEMATICS
PROJECT

*Please post this where others can see it
(with proper permissions, of course!)*

the **PRISON MATHEMATICS PROJECT**

A NON-PROFIT

program supporting inmates
interested in mathematics

**No background
necessary!**

WHAT

Your **mentor** will craft an
individualized plan to help you
achieve your learning goals.

WHO

You'll be **paired** with a
mathematical educator
chosen to fit your level and
whose primary purpose is to
help you **succeed**.

HOW

Communicate through
letters and/or **e-messages** in
which you can discuss
topics, ask questions, engage
with interesting problems
and give or receive helpful
feedback.

WHY

To provide reentry
assistance and broaden
future employment or
educational opportunities
through academic exposure
and improved **confidence**
and **communication** skills.

If you are ready to commit to a life of desistance
from crime through the study and exploration of
mathematics, please write to us at:

Prison Mathematics Project
10810 N. Tatum Blvd Ste 102-998
Phoenix, AZ 85028

Please include:

- a little about yourself
- your mathematical goals
- a brief description of your current mathematical level
- your full mailing address

prisonmathproject.org
PMP@pmathp.org



PMP

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