PMP

PRISON MATHEMATICS PROJECT

THE PRISON MATHEMATICS PROJECT NEWSLETTER

FALL 2023 – ITERATION 7

IN THIS ITERATION

Welcome to our 2023 Fall Iteration of the Prison Mathematics Project newsletter. We celebrate inclusivity and diversity in the math community by reaching all lovers of mathematics, whether incarcerated or not.

Our writers, editors and contributors from around the globe have designed this newsletter to share with you our deep appreciation for the wonderful world that can be discovered through the study and exploration of mathematics. We connect prisoners who are dedicated to change with mathematical mentors, and through this process we introduce them to a community that provides an essential for rebuilding their lives during their incarceration.

The aim of the Prison Mathematics Project is to provide opportunities for our participants to experience a new lifestyle, a new culture that leads to human flourishing through the transformative power of a passion for mathematics.

PMP

PRISON MATHEMATICS PROJECT

The Prison Mathematics Project is a non-profit program supporting inmates interested in mathematics. No background necessary!

Who?

You'll be paired with a mathematical educator chosen to fit your level and whose primary purpose is to help you succeed.

What?

Your mentor will craft an individualized plan to help you achieve your learning goals.

How?

Communication occurs through letters and/or emails in which you can discuss topics, ask questions, engage with interesting problems, and provide helpful feedback.

Why?

To provide reentry assistance and broaden future employment or educational opportunities through academic exposure and improved confidence and communication skills.

If you are ready to commit to a life of desistance from crime through the study and exploration of mathematics, then please tell us a little about yourself, your mathematical goals, and a brief description of your current mathematical level. Include your full mailing address and mail your letter to the address below:

Prison Mathematics Project,
10810 N. Tatum Blvd Ste 102-998, Phoenix, AZ 85028

PrisonMathProject.org

INSIDER INSIGHTS

In this column, we publish feedback from PMP participants, mentors and volunteers. Do you have something to say that will educate, encourage, entice or entertain your fellow math students? Drop us a line at the address on the back page, or if you have access to email, send a message to PMP@pmathp.org. We welcome your input.

We received this letter from participant David Morales Zenquis

Greetings,

I hope this letter finds everyone in good spirits. I simply wanted to notify, to any staff interested, that in the federal carceral system ("Federal Bureau of Prisons") we now have electronic tablets (SCORE 7c tablet by Keefe Group, LLC).

By factory, they come with "Khan Academy", a public domain offline library, and many other functions. Unfortunately, we've been denied access to these. When we ask about this, the facility says the tablets' content is not up to them but to the FBOP offices. The manual from the tablets says that content acceptance is per individual facility; I fear they'll keep playing hot potato at our expense.

Thus, I am contacting all organizations that help inmates to exhort everyone to write to the Attorney General and the Director of the FBOP regarding volunteer services (pursuant to BOP policy "Program Statement 5300.22") and partnerships with organizations for the rehabilitation of all inmates (pursuant to "First Step Act"'s amendment of a statute, that orders them to seek out these organizations or make a compromise with organizations seeking them). They have neglected the use of these policies and laws.

If we can show the FBOP and the AG exactly how many people are willing to be involved in the rehabilitation of inmates nationwide, a new standard of treatment can be set. The advent of the tablets can be used for this, just as well as any other method. But unless society voices out their concern, nothing will change.

So I am writing to ask you to please communicate your desire and concern to these entities. To please tell other organizations to reach out also; the more people talk, the more opportunity we have for change. If your organization struggles with sustainability, this is your chance to make a difference. The fight has to be from both sides of the fence.

The relevant information you can look up for yourself is:
1) FBOP Program Statement 5300.22, "Volunteer Services"
2) Public Law "First Step Act of 2018", amendment to title 18 United States Code section 3621 (h).

Please, write, and share this with any other organization or individual volunteer. Thank you for your attention to this matter, and I hope you have a nice day.

David Morales Zenquis

This article about the PMP first appeared in *Scientific American*'s web edition on June 2, 2023, and is reprinted here with permission.

Pioneering Advanced Math from Behind Bars

Math research gives meaning to years spent in prison

By Amory Tillinghast-Raby

Three years ago Christopher Havens, who has been serving a prison sentence of more than two decades for murder, published a discovery in number theory from his cell. A significant class of fractions, he and three co-authors showed, often maintains a regular structure after being transformed algebraically. Havens's achievement was singular in another respect: he did not have access to computers, which mathematicians commonly program to tackle aspects of such calculations, so he painstakingly pieced his research together by hand.

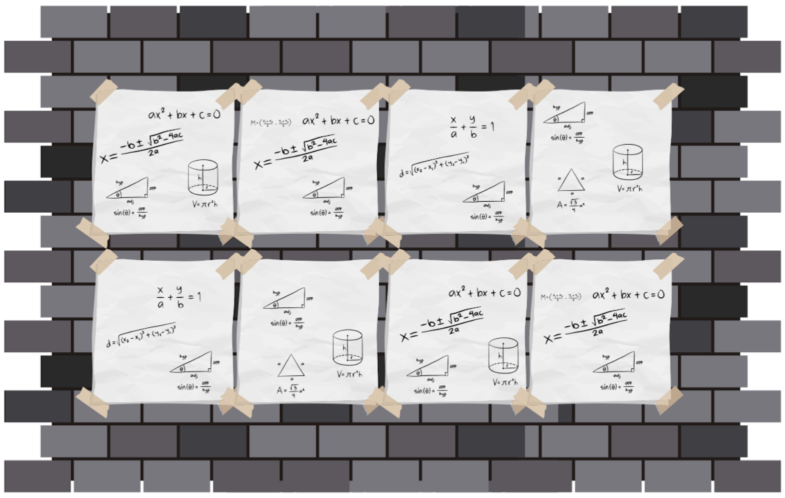
Now a nonprofit co-founded by Havens has invented a computational programming platform built around one of the few technologies that people in prison do have access to: highly restricted, text-only e-mail. And as this facility begins to offer new opportunities, more and more incarcerated people are working on advanced mathematics to give meaning to their years behind bars.

Havens, who dropped out of high school as a sophomore, started studying mathematics in solitary confinement. "It brings out the worst in a lot of people," he says. "Right above you, you got this fluorescent light that never shuts off, not even to go to sleep. You got these guys screaming. There [are] these guys that'll stay up and just kick the wall." To escape the indistinguishable days and nights, Havens began solving math puzzles: first Sudoku and then packets of algebra problems that a prison employee slipped under the door of his cell. "I would get lost in it for days and days and days," he says. "I would dream about it." By the end of his months in "the hole," as people in prison call solitary confinement, Havens says, he was "knee-deep" in calculus and dipping his toes into the field he would ultimately publish in: number theory, the study of integers and the relationships among them.

But even out of solitary, teaching yourself mathematics in prison means getting stuck—not just on a problem but also on where to look for the solution. "Imagine you don't have a professor or anything," says James Conway III, who is studying measure theory—an extension of intuitive ideas about length, area and volume—from Ohio's death row. "You're on your own." So after being released from the hole, Havens wrote to a journal published by Princeton University and the Institute for Advanced Study in Princeton, N.J., asking for a mathematician to correspond with. A few months later a group of researchers wrote back from Turin, Italy, first with guidance and eventually, as Havens's years of mathematical exploration advanced him to the verge of discovery, with the question behind his first paper: How is a continued fraction f transformed by the operation $(af + b)/(cf + d)$?

Continued fractions look like mathematical *matryoshka* dolls, with one nested inside another, which is inside another, and so on, in a series. In this case, each doll is an integer added to a fraction whose denominator is the next smallest figurine in the sequence.

While a "universal" solution to the transformation question the Turin researchers sent Havens has yet to be discovered, Havens found formulas for a specific class of transformations in the patterns of continued fractions he calculated by hand. These long chains of fractions could stretch out across 15 feet of notebook pages wallpapering his cell. It "took over two years to actually do the math," Havens says.



For incarcerated people, days of tackling 21st-century problems with a pen and paper, however, may be coming to an end. Havens co-founded a national nonprofit, the Prison Mathematics Project (PMP), to help other people in prison overcome the challenges of studying mathematics there. Advised by Amit Sahai, a computer scientist at the University of California, Los Angeles, the project has already paired 171 incarcerated people in 27 states with a mentor to work on topics from combinatorics to abstract algebra. One participant, Travis Cunningham, is preparing to submit his research in mathematical physics for publication. Most recently, the project has developed a system to let incarcerated mathematicians write computer programs using only the rudimentary "e-mail" system available to people in prison.

Cranking out hundreds of rote calculations per second, computational programming is an invaluable tool for solving problems across diverse disciplines of higher mathematics. For incarcerated people to write computer code without computer access, the Prison Mathematics Project's programming platform, called the PMP Console, acts as a relay. A person in prison e-mails code from a tablet or kiosk to the console, a cloud-based system extracts and runs the program in an isolated digital environment, and the results are automatically returned.

Havens has already test-driven the console in new work with Carsten Elsner, a mathematician at the University of Applied Sciences for Economics in Hannover, Germany. These latest projects are primarily driven by a specific continued fraction whose nested integers form the sequence $1, 2, 3, \ldots$, Elsner says. "We have given [this fraction] the German name *Zopf*." The name, which translates to "braid," comes from a conjecture Elsner and Havens are looking to prove: that calculating the greatest common factor among fragments of increasingly precise approximations of *Zopf* produces a twisting pattern where the sequence $1, 2, 3, \ldots$ alternates with a sequence of ones: $1, 1, 1, 2, 1, 3, 1, 4, \ldots$

$$Zopf = 1 + \cfrac{1}{2 + \cfrac{1}{3 + \cfrac{1}{4 + \ddots}}}$$

But the name holds a symbolic meaning as well. In German folklore, a traveling nobleman falls into a swamp, sinking deeper and deeper until the muck threatens to swallow him whole. To escape, he lifts himself out of the bog with his own braid. Elsner "suggested '*Zopf*' because of how the numbers twist around the real line and how, in a sense, my life took a similar twisting journey via mathematics," Havens says. Havens "dealt intensively with [*Zopf*] at the beginning of his mathematical studies," Elsner adds. "He tried to communicate his results to other mathematicians and thus start the beginning of a better life."

Although prisoners are interested in the PMP Console—"I'm definitely going to go down that road if I can," Conway says—the system faces significant obstacles to its widespread use in U.S. prisons. Sending an e-mail in prison can cost up to 50 cents, but inmates only earn, on average, a maximum of 52 cents per hour. And if a prisoner can afford to send code to the console, their message still might never be delivered. "[Prisons] have these rules, which are perfectly reasonable, that you can't send encoded messages," Sahai says. "And of course, what they mean by that is enciphered messages." As Sahai recalls, however, the Prison Mathematics Project was told that "according to the dictionary, you know, computer code is code."

Help may be coming. Securus Technologies, a major prison-e-mail provider, is "reviewing the possibility" of incorporating the console into its approved education platform, which includes free "e-message exchanges between students and instructors," says Jade Trombetta, a Securus spokesperson.

But ultimately, whether or not the console sees a broad adaptation, the Prison Mathematics Project isn't a tech company—it's an anchoring point for prisoners weaving their own mathematical lifelines. "Until I started studying mathematics, my life had just been chaos and destruction," says Cunningham, who is serving time for a fatal drunk driving accident. "When I got my first text on partial differential equations, I learned what love is."

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}, \quad k > 0 \quad \left(\begin{array}{c}\text{The Heat Equation}\\ \text{is a PDE}\end{array}\right)$$

Over the past six years, Cunningham developed his initial "love" of partial differential equations into original research on scattering theory, a framework for describing the effects of collisions among free systems such as between particles or waves. His work, which has been guided by a Massachusetts Institute of Technology–trained mathematician, finds new detail about how "resonances"—the physically significant afterimages of a collision involving a particle or wave—are distributed in a simplified quantum mechanical setting. Mathematics "has changed everything for me," Cunningham says.

To Havens, that transformation forms the core of justice. "Justice is not what happens after a person who committed a crime serves X amount of years," he says. "Justice happens when you begin to fix what led you [to prison] in the first place." And although some debts—"infinite debts," as Havens calls them—can never be paid in full, more and more people in prison are turning to mathematics to lift themselves out of the swamp.

THE PRISONER'S DILEMMA

From the Problem Warden

I am delighted to be part of the Prison Mathematics Project through The Prisoner's Dilemma, *as its (honorary) "Problem Warden." My love of mathematics – and especially sharing the joy it can bring – have been a part of almost everything I've done as a professor, financial analyst, parent, founder of the National Museum of Mathematics, author of the Studio Infinity blog, and the most recent ex-editor of* The Playground *(the problem column of* Math Horizons, *a magazine chronicling the world of math for the Mathematical Association of America). I look forward to seeing all of the new methods and creative questions that you come up with as we face the many dilemmas to come, together. And don't worry, as a one-time specialist in mathematical logic, this Warden will keep a sharp eye out for any infractions in your reasoning!*

– Glen Whitney

Glen Whitney, AKA The Problem Warden

We're in the middle of our conversion from publishing solutions in the iteration after a problem appears to publishing them two iterations later. So you won't see many solutions in today's column, but that doesn't mean we haven't gotten them. Thanks for all of your submissions so far, and keep them coming! We have some for every Dilemma up to D12, but can always use more (lots of times different people have different solutions that bring out new aspects of a problem). So far we've received nothing on D13 "The Experbola," so get cracking on that curve – even just a partial attempt would be welcome. In the meantime, we've got four fresh Dilemmas for you to ponder.

D14: Flip Functions

Contributed by Yagub Aliyev, ADA University, Azerbaijan

Call a function f which maps the real numbers to the real numbers a *sign-flipping function* if for all $x, y \in \mathbb{R}$,

$$(x-y)(f(x)+f(y)) = (x+y)(f(x)-f(y)).$$

For example, the identity function $i(x) = x$ is a sign-flipping function because the equation above reduces to $(x-y)(x+y) = (x+y)(x-y)$ which is true by commutativity of multiplication. Are there any others? Find all sign-flipping functions.

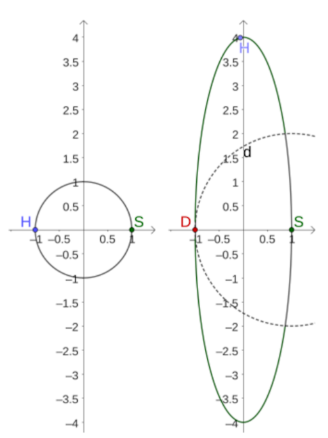


Figure 1: Avoiding the smell at S by hiding at H.

D15: Eccentric Repulsion

Contributed by Paul Morton, PMP Participant

Paul is back with another elliptical quandary. Suppose you are constrained to a circular track, which we will model as the unit circle in the plane, and there is something smelly at one point on the track, say where it intersects the x-axis at the point $(1, 0)$. If you want to get as far away as possible from the smell, clearly you should run to the diametrically opposite point on the track, namely the point $(-1, 0)$. Now suppose you change the shape of the track by stretching it in the y-direction into an ellipse. If you stretch it far enough, then the diametrically opposite point will no longer be the farthest away: as shown in Figure 1, when you have stretched it by a factor of four, most of the ellipse lies more than two units away (the portion shown in green).

What is the *most* you can stretch the track so that the opposite point $(-1, 0)$ remains the farthest place to hide from the smell? (Paul originally asked this question in terms of the *eccentricity* of the elliptical track: if the circle is stretched by a factor of $a > 1$ in the y-direction, then the eccentricity of the resulting ellipse is $e = \sqrt{1 - 1/a^2}$. You may phrase your answer in terms of either the eccentricity e or the stretch factor a.)

D16: Five Choices

Contributed by Michael Levitan, Villanova University

You are at a crossroads with five roads leading away from it, and you don't know which direction to go. You would like to choose randomly, with equal probability for each road. If your only way of making random choices is to flip a *fair* coin (one that could come up heads or tails with equal probability), you could assign one of the roads to each of the sequences HHH, HHT, HTH, HTT, and THH. Those outcomes are all equally likely, so you know that if you flip three times and choose the corresponding option, each one of them will have an equal chance. The problem is that there are three outcomes that are not assigned to any road, so if one of those three comes up you will have to try again. Therefore, it could take you arbitrarily long to make your decision, if you are unlucky and keep getting one of the unassigned outcomes.

Figure 2: Which way to go? (Image credit: Dall-E)

You don't have all day to stand around, so you'd like a better method. But it turns out that people have proved there is **no** process with a single fair coin that is guaranteed to finish in a finite amount of time and choose one of five options with equal probability.

a) Suppose you could have a single *unfair* coin that would come up heads with probability $p \neq 1/2$ (and tails with probability $1 - p$). Is there a p you can select and a procedure for using this unfair coin to choose one of the roads, each equally likely, that is guaranteed to complete in a finite number of flips?

b) If so, can you determine the smallest number n of flips that could be used, and what probability p you want the coin to have to be able to decide in n flips?

c) Now suppose you are allowed to have more than one type of coin, each one marked with the probability that it comes up heads. What's the fewest flips you would need to choose your direction? How many coins do you need, and what's a set of probabilities for the coins that lets you do it in that minimal number of flips? If your set contains $1/2$, that's OK; but is there also a way to achieve that minimum with all unfair coins?

D17: Transrotation
Contributed by Akiva Weinberger, Yale University

Often you can slice up one shape and reassemble the pieces to make a different shape, like the famous square-to-triangle dissection shown in Figure 3. Notice that in this rearrangement you are forced to rotate some of the pieces when you put them back together. Sometimes, though, you can do the assembly by only *translating* the pieces, i.e. sliding them without rotation, as in the square-to-cross dissection shown in Figure 4.

Is it possible to "rotate by translation?" In other words, can you dissect a square into finitely many pieces, and translate each of them without any rotation, to reassemble them into the shape of a square that has been rotated (by some angle other than a multiple of 90°)? Note that although this problem was suggested to the *Prisoner's Dilemma* by Akiva, it was apparently first posed by Hugo Hadwiger way back in 1950.

Figure 3: Dissecting a square into a triangle.

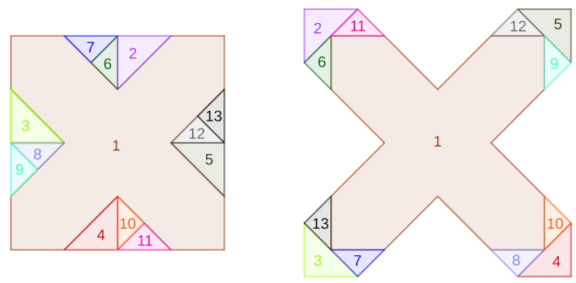


Figure 4: Sliding pieces of a square to make a cross

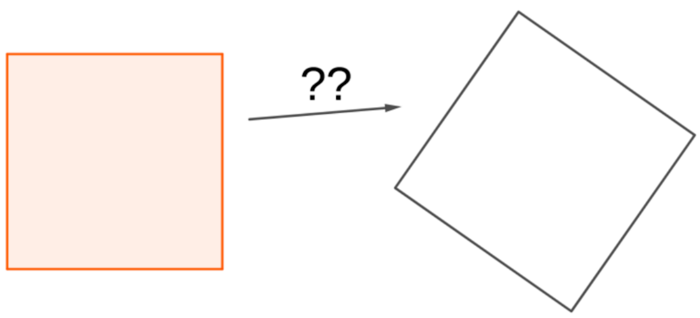


Figure 5: Can you rotate by slicing and sliding?

SECOND HEARING

In this portion of the column we revisit prior dilemmas, to share solutions that readers have come up with, or report on their status. This time, we have just one part of one problem we're wrapping up, but we'll restate it in full and summarize the whole thing for a hopefully satisfying finish.

D4: Four Operations (Re)-revisited
This problem challenged you to describe the long-term behavior of four different sequences, each of which used a different one of the four basic operations on the previous two terms to determine what to add to the previous term. Here "long-term behavior" could mean an expression for the nth term that just involves n, not the previous terms. If there doesn't seem to be a closed form, it could be a formula that works when n is bigger than some number, but might be off for the first few values. Or sometimes you can find a formula that is never exactly right, but gets closer and closer as n gets large. All else failing, it could be one formula that's always less than the sequence and another that is always greater – and hopefully you can get those two "bounding" formulas as close to each other as possible. Each of the sequences started with first term 1 and second term 2, and thereafter were defined by the following four recurrence relations:

$$\begin{aligned}
a_{n+2} &= 2a_{n+1} + a_n - 1 \\
s_{n+2} &= 2s_{n+1} - s_n + 1 \\
m_{n+2} &= m_{n+1}(m_n + 2)/2 \\
d_{n+2} &= \lceil 2d_{n+1}/d_n \rceil + d_{n+1}
\end{aligned}$$

(note we've changed the original letters used by each series to remind ourselves they are based on addition, subtraction, multiplication, and division). The original problem also asked which sequence ultimately grows the most slowly?

To recap what had happened before, submitters showed that as n grows large, $a_n \to \frac{(1+\sqrt{2})^{n+1}-1}{4}$. The subtraction-based sequence is given exactly by $s_n = 1 + n(n-1)/2$. And the division sequence ends up satisfying $d_n = 3n+1$ for $n > 4$. That left just the multiplication sequence untamed.

Into this breach, PMP participant William Keehn threw himself. He offered the insight that when dealing with a lot of multiplication, it's often helpful to take logarithms. In particular, because of the occurrences of the number 2 in the recurrence for m_n, William decided to use base 2, and set $l_n = \log_2 m_n$ (so that also $m_n = 2^{l_n}$). He calculated that the sequence of l_n values starts

$$0, 1, 1.5849625\ldots, 2.5849625\ldots, 3.90689\ldots,$$
$$5.90689\ldots, 8.99435\ldots, 13.9485\ldots,$$

and so on. These numbers are looking very reminiscent of the familiar Virahanka-Fibonacci (VF) sequence $v_1 = 1, v_2 = 1, v_3 = 2, v_4 = 3, v_5 = 5$, etc., and that resemblance turns out to be no surprise: if we take log base 2 of both sides of the recurrence for m_n, we get $l_{n+2} = l_{n+1} + \log_2(m_n+2) - 1$. We still would like to get rid of the remaining m_n in this expression, so we use a technique known as Standard Expert Trick #367: Add zero. The key to using this trick is to find the right term to both add and subtract (in effect, adding zero) to simplify this expression. In our case, we would like an l_n term, so we try adding $0 = l_n - \log_2(m_n)$ to get $l_{n+2} = l_{n+1} + l_n - 1 + \log_2((m_n+2)/m_n)$. As $(m_n+2)/m_n = 1 + 2/m_n = 1 + 2^{1-l_n}$, we convert back to natural log to take advantage of the series expansion of $\ln(1+x)$ near $x = 0$:

$$\begin{aligned} l_{n+2} &= l_{n+1} + l_n - 1 + \frac{1}{\ln 2}\ln(1+2^{1-l_n}) \\ &= l_{n+1} + l_n - 1 + \\ &\quad + \frac{1}{\ln 2}\left(2^{1-l_n} - 2^{2-2l_n} + 2^{3-3l_n}/3 - \ldots\right). \end{aligned}$$

We can see from this expression and the initial values above that the l_n are growing rapidly, like the VF series, so all of the terms in parentheses become large negative powers of two, and quickly become entirely negligible. In other words, as n grows, $l_{n+2} \to l_{n+1} + l_n - 1$. More directly, we observe that $l_4 > v_2 + 1$ and $l_5 > v_3 + 1$, and that if $l_{n+1} > v_{n-1} + 1$ and $l_n > v_{n-2} + 1$, then $l_{n+2} > l_{n+1} + l_n - 1 > v_{n-1} + 1 + v_{n-2} + 1 - 1 = v_n + 1$. So by induction, we conclude that for all $n > 3$, $l_n > v_{n-2} + 1$. (Moreover, using the estimates above we can actually show that $l_n \to v_{n-2} + 1$ as n grows.) In any case, converting back to the original series, we have our lower bound (which turns out to be very close in the long run):

$$m_n > 2^{v_{n-2}+1}.$$

At long last, this puts us in a position to answer the final question of D4: which sequence grows the slowest? Looking at the expressions we've derived over the past year, we see that a_n grows exponentially. Multiplying out the exact formula $s_n = n^2/2 - n/2 + 1$ shows that it grows quadratically. Except for a few terms at the beginning, d_n is linear. And since the Virahanka-Fibonacci sequence itself grows exponentially, m_n is the monster of the lot and grows double-exponentially. Of these, division produces the slowest-growing sequence. To be sure, PMP participant Robert Noll sent a letter pointing out (a) guessing that division leads to slow growth would have been a good bet right from the start, and (b) we didn't need an asymptotically correct formula for m_n to see that it couldn't be the slowest growing: It is very easy to prove the much weaker lower bound that $m_n > 2^{n-2}$, which already makes it grow faster than any quadratic function, let alone linear.

Nevertheless, in figuring out at least decent approximations for each of these series, and exact formulas where possible, we got to encounter many different problem-solving techniques. Hopefully this journey has been more interesting than if we just tried to find the slowest-growing one right away. That's a lot of mileage from just the four basic arithmetic operations! ■

CLEANING IT UP

Every enterprise sometimes makes its messes, and nobody should be ashamed to admit their mistakes – or be afraid to fix them. In this occasional section, the Prisoner's Dilemma *corrects any issues that slipped through in earlier iterations.*

Credit Where Credit is Due

The editors would like to acknowledge that Matthew Helmer (Pacific Lutheran University) submitted a lovely solution to D6 "Rectangle Pinwheel," in addi-

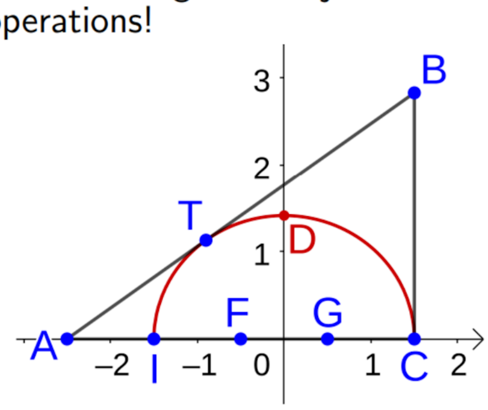


Figure 6: Semiellipse inscribed in right triangle CBA so that its foci and other vertex cut a leg in four equal parts

tion to the other solvers already credited. Similarly, D8 "Mullipse?" was also completed by Christopher Bistryski, a PMP participant from Monroe, WA. Congratulations to both solvers, and our apologies for the earlier oversight.

D9: Semi Inellipse

Although we already presented a solution to this problem, PMP participant Paul Morton noticed that we had dropped an exponent that made the equations very confusing to follow (not to mention just plain wrong). Looking into that error led us to discover another, and then another... So it seems best to just run the entire solution again, this time hopefully with many fewer mistakes.

As shown in Figure 6, semiellipse CTI is inscribed in right triangle CBA so that it is tangent to hypotenuse AB at T, and so that points I and the two foci F and G of the ellipse cut leg AC of the triangle into four equal segments. What is the ratio of the lengths of hypotenuse AB and leg AC, i.e., AB/AC?

For convenience, since segment AC is divided into four equal parts, we introduce coordinates scaled so that $AC = 4$. We place the origin at the center of the (semi)ellipse, which is to say the midpoint between the foci F and G. Thus the equation of the ellipse will have the standard form $x^2/a^2 + y^2/b^2 = 1$ for some constants a and b giving the positive x- and y-intercepts of the ellipse. We've included this coordinate system in Figure 6.

The coordinates of some of the points are immediate:

$$A(-5/2, 0),\ I(-3/2, 0),\ F(-1/2, 0),$$
$$G(1/2, 0),\ \text{ and } C(3/2, 0)$$

Since C is the x-intercept of the ellipse, we have $a = 3/2$. We also added the y-intercept $D(0, b)$ in Figure 6. Since $FC + CG = 3$, we have by the definition of an ellipse that also $3 = FD + DG = 2\sqrt{b^2 + 1/4}$, from which we solve for $b = \sqrt{2}$. Now suppose the coordinates of point B are $(3/2, h)$. Then the equation of line AB is

$$y = \frac{h}{4}\left(x + \frac{5}{2}\right).$$

Substituting this value for y into the equation of the ellipse to solve for the x-coordinate of point T (since the equations of both the line and the ellipse must be satisfied at point T), we get

$$\frac{4x^2}{9} + \frac{h^2}{32}\left(x + \frac{5}{2}\right)^2 = 1.$$

We can write out the rightmost square and multiply both sides by $1152 = 9 \cdot 32 \cdot 4$ to obtain the quadratic

$$(36h^2 + 512)x^2 + 180h^2x + (225h^2 - 1152) = 0.$$

However, it is given in the problem that the line AB is tangent to the semiellipse, so they only intersect in one point. Therefore, this quadratic must have only one real root. But by the quadratic formula for $Ax^2 + Bx + C = 0$, namely $x = \left(-B \pm \sqrt{B^2 - 4AC}\right)/2A$, there are two roots unless $B^2 - 4AC = 0$. In other words, it must be the case that

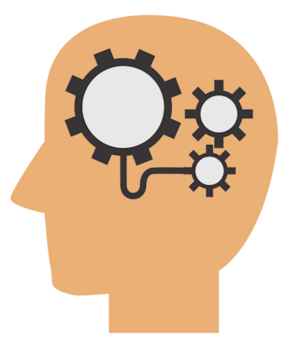
$$(180h^2)^2 = 4(36h^2 + 512)(225h^2 - 1152).$$

Again multiplying out, this equation simplifies to $h^2 = 8$, amazingly enough. Armed with this information, it is easy to compute that $AB = \sqrt{16 + h^2} = 2\sqrt{6}$, so $AB/AC = \sqrt{6}/2$. ∎

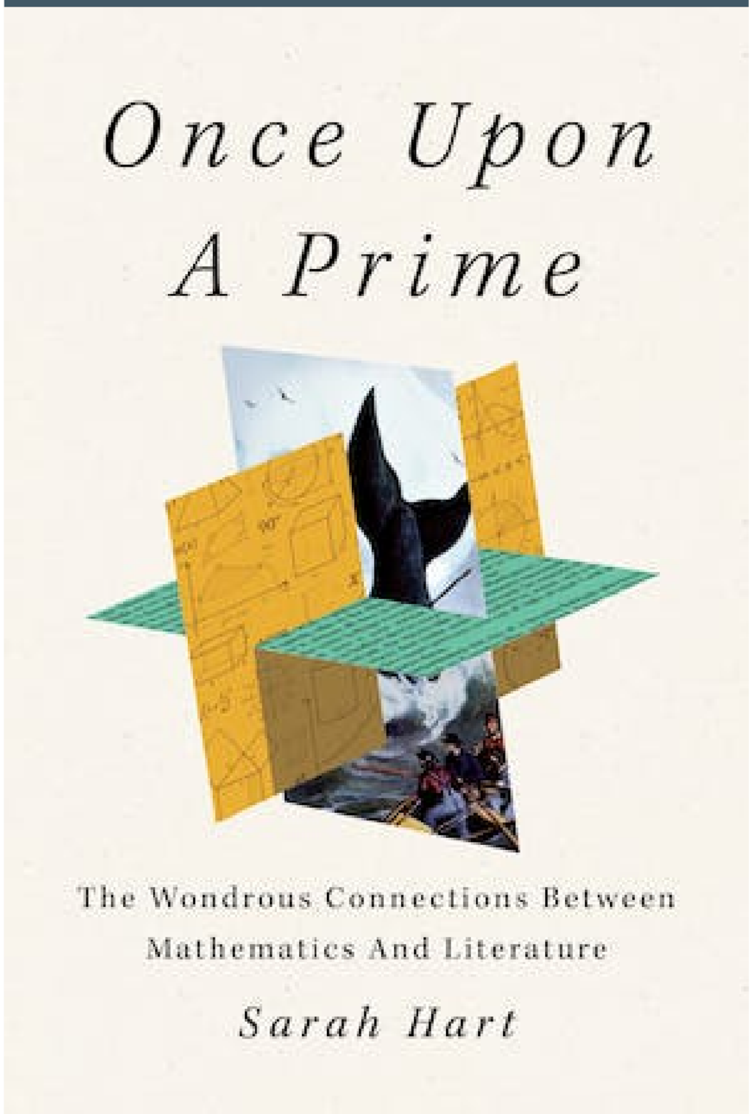
Submission Guidelines

Solutions to problems published in *The Prisoner's Dilemma*, and proposals for new Dilemmas, are welcome. For solutions, please clearly indicate the Dilemma number being solved. If a problem has multiple parts, you may submit solutions to any individual part or parts. Solutions to the Dilemmas in this newsletter must be received by the deadline of **2024 March 1**, and they will appear in the iteration *after* the next one. Dilemma proposals will be considered on an ongoing basis. All submissions should be addressed to Glen Whitney/Prisoner's Dilemma either by email at dilemma@pmathp.org (in which case PDF format is preferred, if possible, although any reasonable format will be accepted), or by mail at:

Prison Mathematics Project
Glen Whitney/Prisoner's Dilemma
10810 N. Tatum Blvd, Suite 102-998
Phoenix, AZ 85028



BOOK REVIEW CONTEST



ANNOUNCING OUR NEW READING RECOMMENDATION!

Our featured book is *ONCE UPON A PRIME: The Wondrous Connections Between Mathematics and Literature*, by Sarah Hart.

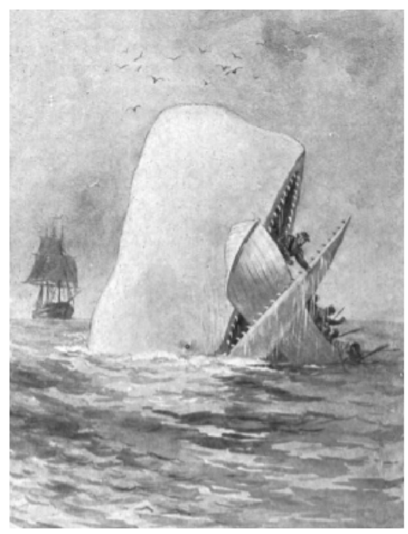
This book was brought to our attention by one of our PMP board members, Norton Ewart, who recommends it with enthusiasm:

> Hello PMPeople,
>
> If I may interrupt your $n$day (where $n \in$ {Fri, Sat, Sun, Mon, Tues, Wed, Thurs} depending on when you read this, (I'm writing it on $n =$ Fri) I have discovered a book I hope you can get your hands on and read as soon as possible, because each day of delay is doomed to be a less-rewarding day of your life. The book is "Once Upon A Prime: The Wondrous Connections Between Mathematics and Literature" by Sarah Hart.
>
> It has three parts:
>
> - Mathematical Structure, Creativity and Constraint
> - Algebraic Allusions: The Narrative Uses of Mathematics
> - Mathematics Becomes the Story
>
> Not every chapter will please everyone, but everyone will be pleased by at least one of the ten chapters. I would love to know what our participants think of it.
>
> \- Norton Ewart

In 2013 Sarah Hart became one of only five women Mathematics Professors under the age of 40 in the UK. Her book illustrates how math and literature are far from polar opposites. She connects the dots between the two – and shows how understanding the connection enhances our appreciation of both.

For instance, did you know that George Eliot (the novelist Mary Ann Evans, using a male pen name, which is what a lot of women writers had to do to get published in the 1800s) recommended taking "a dose of mathematics every day"? That Leo Tolstoy used calculus in *War and Peace*? Or that Lewis Carroll sprinkled math puzzles throughout *Alice's Adventures in Wonderland*? (He was a mathematician.) Were you aware that Moby-Dick is awash in "lovely juicy mathematics"?

Once Upon a Prime examines the hidden patterns in these and other books, some well known, others more esoteric, revealing layers of beauty and wonder—and teaching a little math along the way.



Moby-Dick, illustrated by Augustus Burnham Shute, from an 1892 edition of the novel. It's not just the math that's juicy!

"By seeing mathematics and literature as complementary parts of the same quest to understand human life and our place in the universe," Hart writes, "we immeasurably enrich both fields." She asserts that poetry is "simply the continuation of mathematics by other means."

According to the New York Times Review of Books:

"Hart couldn't be better placed to revisit the relationship between mathematical and literary study. She's an accomplished group theorist at the University of London and the current Gresham professor of geometry, a position so old that its first occupant invented long division, and whose mission has always involved mathematical outreach to the general public."

She is funny, even as she smashes "the beguiling fantasy that scientists, and especially mathematicians, are driven by pure reason, that cleverness can get you out of any fix, and that everything can finally make sense if you can just ramify the ninth-dimensional asymptotes over a tangential vector field. Sadly, you can't, first, because life isn't like that, and second, because I've just made up all those phrases, so they are meaningless."

Here's how the review contest works: You read the featured book and tell us what you think of it. If your review is deemed by our panel of judges to be the most inspiring to potential readers, it will be published in a future edition of the newsletter and on the PMP website.

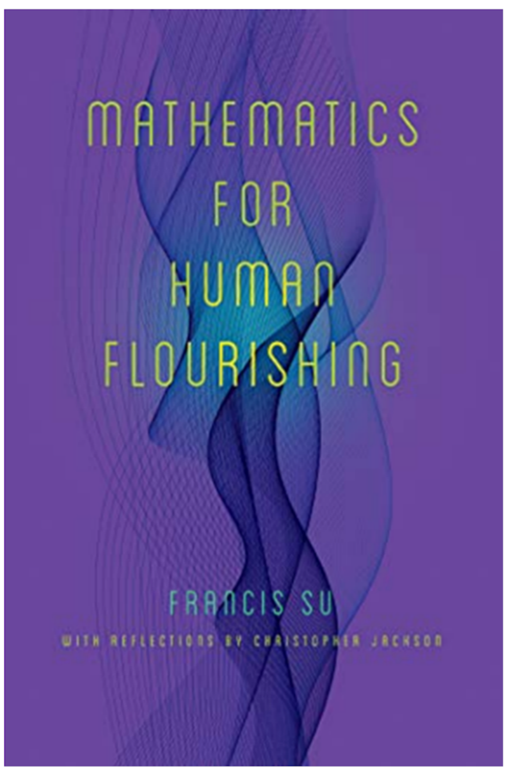
If you can't obtain the featured book on your own, please request a free copy from the PMP (address on the back). Once you've read the book, send your review to us via snail mail, or email it to **pmp@pmathp.org**. There is a catch: we send you the book, you send us a review. Fair? Reviews must be no longer than 1,000 words and may be edited for length or clarity. All submissions for *Once Upon a Prime* must be received by **February 29, 2024**.

And an Extension!

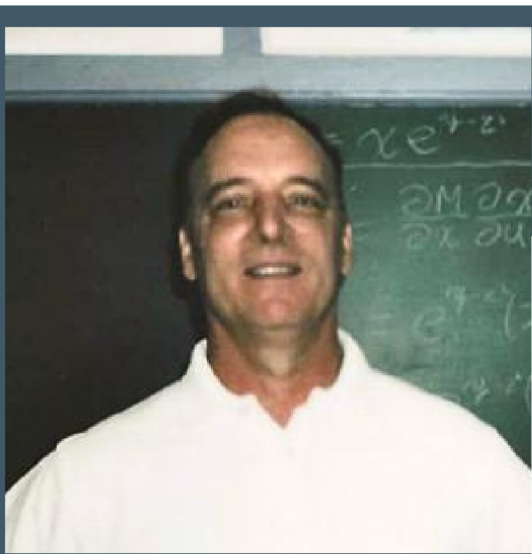
Our last feature was *Mathematics for Human Flourishing*, by Francis Su. James Tanton of the Global Math Project calls this book, "Perhaps the most important mathematics book of our time."

It's not too late to participate! We have extended the deadline—submissions will be accepted until **October 30, 2023**.

We warmly invite you to read this lyrical ode to the connection between mathematics and our common humanity—and then submit your review.



At its core, *Mathematics for Human Flourishing* is a meditation about what it means to be human. Su explains how the study of mathematics can meet many of our natural human desires and build virtues that help us flourish, even in the most difficult circumstances. Su's incarcerated friend Christopher Jackson, who discovered his own love of math in prison, exemplifies this truth. Through the arc of the book, in Jackson's letters and Su's narrative, we see how our humanity reveals that everyone is a 'math person.'



Paul Morton

Claire Finlayson

PARTICIPANT SPOTLIGHT

by Claire Finlayson

Well, someone has to keep these math geeks in line and jump on every little spelling mistake...

I'm a writer from BC, Canada, so don't try to bust me for using British spelling, like "cheque" and "colour," okay?

I am a PMP superfan, and my rudimentary math skills have not proven to be an impediment to my participation in this wonderful organization. It's the unique, true-life stories of people that interest me. So I write profiles of participants, volunteer mentors and others involved in the program.

If there's someone you'd like me to spotlight, here's how you can contact me:

www.clairefinlayson.com

claire@pmathp.org

Prison Mathematics Project,
10810 N. Tatum Blvd Ste 102-998
Phoenix, AZ 85028

Note: This interview took place in July 2023 while Paul was serving his last remaining weeks in prison, but he'll be a free man living in the aptly named Liberty, NY, by the time you read this.

CF: Hi, Paul. So your last day in prison is coming up quick. How are you feeling about it?

PM: My release date is technically Saturday August 5th, but NY does not release people on Fridays, Saturdays, or Sundays, so I am being released on Thursday, August 3rd. Kind of makes up for the two leap years I've spent here...

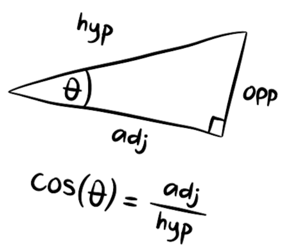
I am definitely glad this prison thing is over with, but I can't say I have visions of unicorns and rainbows. I have a lot of hard work ahead of me.

CF: Yes, and that's in addition to the hard work you've already done. What are your main concerns at this point?

PM: I guess I am apprehensive about things like finding a decent job, making new friends... basically starting my life over again. However, I have never had a hard time finding work. I think my biggest concern is that I am always going to have a "past" that many people are going to have a hard time accepting. I am at an age where starting a family is not an option, but I would like to grow old with someone. So to answer your question, I think being alone is my biggest concern. As far as post-release goals, other then the immediate ones (job, housing, etc.) I definitely want to continue expanding my knowledge of mathematics, and after I have completed my three years of post-release supervision I am leaving this hell hole called New York and moving out west. I have been doing a lot of research on Wyoming and the panhandle of Alaska.

CF: That will be a fresh start for you. So, about mathematics, I know it has long been a passion of yours, Paul—it's impossible to talk to you and miss that fact!—but can you tell me how you came to be associated with the PMP?

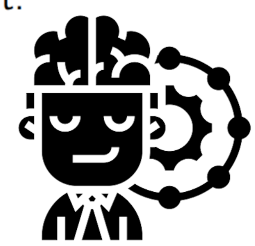
PM: I first learned about Christopher and the PMP from the February 2021 *Popular Mechanics* article that featured him. Somebody here who knows I'm a math junkie gave me the magazine and said, "Hey, Pauly, this guy is like you."



When I saw the title, "*Prisoner in solitary confinement solves some of the world's hardest math problems*," I said, "No, this guy must be more like 'Rain Man.' I can't compete with him." But I read the article and wanted to learn more about PMP, so I wrote to the address and here we are a few years later.

CF: Here we are. There's talk of you continuing your involvement with the PMP after your release. Any substance to that rumor?

PM: I would really like that to happen, but it is up to Christopher and PMP board. I think at some point soon we will be having that discussion. I have many ideas that I think will get the participation rate to increase. Me spending ten years in the slammer gives me some insight as to how prison officials work and how prisoners think. At its heart, PMP is about turning prison time into math time, and as a result participants leave prison smarter, more reasonable in their problem solving skills, and with some sense of accomplishment.



CF: You are already pretty accomplished. I understand you've written a calculus textbook!

PM: Let's be clear on what I wrote. It is not a textbook, it's a book that helps you understand what the textbooks are saying. Calculus can be very intuitively challenging, and textbooks sometimes have a weird way of trying to help you understand something by showing you a proof that is even *more* intuitively challenging. I took about 30 different areas in single variable differentiation and integration that I personally had the most difficulties with, and through step-by-step application I give the reader a clearer picture of the concept where the textbook, by itself, may be too difficult to understand. Ultimately, being self-taught has given me a unique perspective on textbooks that most people don't have. When you are in a classroom setting you have a teacher or professor who helps you fill in the intuition gaps in the texts. I never had anyone to fill in those gaps. It took a lot of hair pulling and sleepless nights for me to fill in those gaps by myself. In my book I give the reader a layman's perspective without all the textbook jargon. I think of my book as ancillary to a calculus text. This project took me just over a year to complete, and I think it was quite the accomplishment considering my surroundings and resources. As far as publishing, I hope that will become a reality. It's a really good book, and besides, I would like something to show for this prison time.

CF: So your book is like a "companion guide" to the textbooks! I really hope you find a publisher, Paul. That would be a nice feather in your cap. And maybe because of you there will be fewer people wearing caps because they sat up all night pulling out their hair...

Is it safe to assume you've always loved math, then?

PM: Well, no, actually. I really didn't have any opinion about math. I didn't like it or dislike it. It was just *there* in the background of life.

As far as aptitude, I never thought about whether I would be good at it or not. It's kind of a funny story, my math journey. I wasn't really trying to learn math. I had gotten my hands on a book called "*Introduction to Physics*." It was an algebra-based physics book and very intriguing, but I couldn't grasp the concepts because I didn't understand the math. Needless to say, my curiosity got the best of me. I asked another inmate who was in the college program at the prison if he could get me an algebra textbook I could borrow for a week or so, just so I could understand this physics book. You can understand why I think this story is funny: I thought I was going to knock off algebra in like a week!

CF: Wait, you mean you can't learn algebra in a week? I'm kidding! Even I know it takes a minimum of two weeks... But seriously, on your way to becoming a physicist, you accidentally became a mathematician. I think something like that happened to PMP founder Christopher Havens, too. He had a thing for astronomy and look where that got him!

PM: Sadly, no, you cannot learn algebra in a week. But anyway, deluded though I was, I was given a copy of "*Beginning Algebra, 10th Edition*" by Lial, Hornsby and McGinnis, and here I am over six years later, a full blown "mathaholic."

CF: One of the stated goals of the PMP is to reduce recidivism by introducing inmates to a new culture and healthy friendships based on a shared love of mathematics. Do you think the study of math under the guidance of a PMP mentor can actually help rehabilitate a person steeped in a toxic culture of gangs and crime?

PM: *Rehabilitation* is an interesting word. If you ask ten different people the meaning of that word, you will get ten different answers. New York State doesn't "rehabilitate" or "punish" anybody—they gave up on that a long time ago. We are simply segregated from society, like a "time out."

It's just that some of us have more "time out" time than others. So back to your question. I never really considered immersing myself in mathematics as a form of rehabilitation. But what I have come to realize is that many people walk through life in search of something that will bring them some sort of happiness or meaning. I was one of those people. I spent many years inside of a bottle trying to erase past hurts and regrets, trying to hide from myself. After about a month into studying math, I noticed that I was starting to look forward to my "math time." As months went by, I started to feel an inner peace; I was experiencing joy as I was learning, and a sense of accomplishment was developing. I was starting to feel better about being me. I know this might sound bizarre to most, as I was just learning basic algebra. Claire, I was married to a beautiful successful woman, lived in a very nice house, had my own business, etc. but I was never really happy or at peace with myself. I have found both inside of the pages of my math books. So if you consider this rehabilitation, that's fine, I think of it more as an awakening. Mathematics has also given me a place to escape the horrors of prison. When I am in my cell with my math book open there could be a tornado coming down the street, a riot in the yard, and a mushroom cloud on the horizon, all at the same time, and I wouldn't know it. After about a year I started calling my math book "my secret garden." Even now, the name seems so appropriate as a description of where I am when I am studying.

CF: Some people enter their own secret garden through writing, dance, painting, gardening or music. There are many portals, but everyone who enters has found something truly special. I love the idea that you've found such a place right there inside the walls of a prison. In that respect, you are privileged. Say, have you seen Christopher in the garden? He doesn't spend a lot of time in prison, either!

You've written an essay about math—more like an ode to the study of math—which we are publishing in this newsletter, so we need to wrap up this conversation to save room for it. What do you have to say to others who might be a little more intimidated than you were, or who might think they're not smart enough to enroll in the PMP program?

PM: What I would say to *anybody* who might want to learn math and may be too intimidated is TRY IT FOR A MONTH. Find a good beginning algebra book and start on page one. If you are already familiar with the material, then consider it a review, but I guarantee you, you will pick up things you have forgotten. After a month you will definitely know whether or not this is what you want to do. Then contact PMP and get hooked up with a mentor. You are not going to be a guru in the beginning, nobody is, but you will eventually be a guru, because you have that same thing that drives the likes of Christopher and myself: *mathaholism*. And you won't stop.

Eventually months turn into years, algebra turns into precalculus, then calculus, linear algebra, differential equations, and beyond. Your cell is littered with math books, thousands of pages of solved problems, equations, and proofs you wrote. Most of your hair has fallen out, and you take immense pride in being called a geek. Then you wake up one morning and realize that you are leaving in a couple of months....it's *over*.....but the math never ends.

In all seriousness, if you have years to do, don't waste them by hanging out in the yard or watching TV. **The fact that you are even reading this newsletter shows that you have some interest.** See if it's for you. You won't regret it.

CF: That's a compelling invitation. We have mentors waiting in a variety of math disciplines and all competencies, but it's a little harder to get info about the PMP into the hands of inmates. So thank you. Can you tell us about your plans post-release to continue your math journey?

PM: Well, I know it may take a little while to get situated, so for the short term I am going to keep doing what I am doing now, which is reviewing my calculus, studying linear algebra and differential equations. After I am settled I would like to take some classes on number theory and discrete mathematics. Those courses will help me with mathematical structure, which I feel I am lacking. I hope to stay involved with PMP in some capacity. As well, I would like to volunteer as a tutor and mentor for teens that are going down the wrong path. I feel that introducing mathematics to a "problem child" might actually be a great way to keep that kid's mind occupied in a positive way, and just might make the difference between him/her going to college vs. prison.

CF: It was a pleasure speaking with you, Paul. On behalf of the PMP team, we wish you a beautiful future and thank you for being such a star participant in our program!

DON'T LOSE YOUR MATH

by Paul Morton

I am soon to be released after doing 10 years 7 months and 20 days on a 13 year sentence. Your release date is what many of us incarcerated consider the most "anticipated day." Yes, I am very excited to get back to life, but also I have a bit of trepidation about what I call losing my math.

My math journey began in early 2017. I started on page one of a beginner algebra textbook and I have been on this road ever since. Lately, I have been spending a lot of time contemplating my future while reflecting on my math journey. In my cell I am surrounded by dozens of math books, each with its own story. I think about how insanely difficult it has been learning mathematics inside a prison cell. However, nothing could stop me from spending hours a day, every day, for years in this self-torturous endeavor. Somebody told me this is the definition of true love: a relentless desire to continue learning and growing—even if it can be painstakingly difficult.

Being self-taught, as most of us are, we are our own students and our own teachers. When we don't understand something, our whole world stops moving until we can figure it out. There is nobody we can ask for help. There is no access to Google or a computer to research answers. All we have is ourselves, and of course our PMP mentors, but sometimes it can be weeks before we get a reply from them. Most people would not consider it a hardship to obtain some fine point pens for graphing, graph ruled composition notebooks, or even a decent calculator. For us, these items are such a luxury that the stars have to align perfectly for us to obtain them. Not to mention having the textbooks we want, or finding solutions manuals to our textbooks that are not insanely expensive. All this is needed just so we can reverse-engineer a solution to get the insight and intuition into the problem or concept we don't understand. I mention all of these hardships to make a point. We are incredibly tenacious people, and the thought of quitting or giving up is not an option for us.

There are times when I am studying a new subject or concept that something I studied before will reappear in a problem, and sometimes I don't get the answer immediately. Even though I know the math, I may not have used it in a while. I have an inside joke between me and myself, and I say out loud "I AM LOSING MY MATH!" Now, I may be rusty in an area, but certainly I am *not* losing my math. Lately though, I wonder, when I leave here and go back to living life outside of my cell and with all the busyness that comes with that, will I have the time and desire to continue with my math? Believe it or not, this is what scares me the most about leaving prison! Math is a tremendous part of my joy, passion and drive. Will I change if I don't have the time to devote to math?

Doing a long-term prison sentence has a unique effect on the human mind. We create our own little universes to live in, so we don't have to confront the reality of prison life. For those of us that built our little universe around the study of mathematics, we have found a joy and solace that others around us don't quite understand. I have noticed that as the years roll by and the more advanced I have become in math, people have begun to treat me differently, including some of the officers. There is a certain kind of respect. They know what I am doing is very difficult and takes an enormous amount of discipline. They also know they could never do what I am doing.

Whether you are aware or not, your growth in mathematics has impacted you in positive ways. You are studying a language that many consider the greatest accomplishment of the human mind, and for the most part you are teaching yourself. This has given you a great sense of accomplishment and has also improved your self-esteem. You can look at things more maturely, and approach situations with real problems-solving skills. Anybody who has studied calculus in any depth has a greater understanding of how the natural world works, and a deeper appreciation for nature than most people will ever experience.

I have made a promise to myself that no matter where life takes me, I will always take my math with me. No matter how busy I am, I will always find time to play in my secret garden, a place where I can go with just a radio and my math book and hide out. No matter what is going on in my life, I will never forget the "math-magic" moments I have experienced—those times when I finally understood a proof, or solved that seemingly impossible equation, the thrill and excitement I felt when I learned something new.

When I am released, I plan on making new friends—math friends. These are like-minded people that I can relate to, have fun with, and learn from, who also have the passion I have for mathematics. I also plan on staying very close to PMP, and helping wherever I am needed.

We are special people. You are a special person. When it is your turn to go out again into the world, promise yourself you will not lose what it is that makes you special. *Don't lose your math.*

THAT IS SO COOL!

Dr. Amy Shell-Gellasch

Dr. Amy Shell-Gellasch is a full time lecturer at Eastern Michigan University. She earned her DA in mathematics from the University of Illinois at Chicago in 2000 and followed that with a post doctorate position at the United States Military Academy at West Point, NY. Her area of research is the History of Mathematics and its uses in teaching. She co-founded and currently chairs the History of Mathematics Special Interest Group of the MAA and is an associate editor of Convergence online journal. Currently she is the Chair of the Michigan Section of the MAA. She conducted research on mathematical devices at the Smithsonian National Museum of American History from 2012-2017.

Parabola Multiplication

Collecting different ways to do multiplication is a bit of a hobby with Christopher Havens and myself. I also teach and enjoy playing with Analytic Geometry (using algebra to answer questions in geometry and vice versa, basically what is now known as Algebra II or Precalculus.) So I was very excited several years ago to discover that there is a method to multiply two numbers using the ubiquitous parabola $y = x^2$. Although I would not suggest using the method to actually perform multiplication, it is more of a very cool connection between parts of elementary mathematics and arithmetic.

How does it work? Pick two numbers, for simplicity lets make them "small". I will use the birth month of myself (October) and my dog (July) - who was referenced but not introduced in my last article. This is Sadie, a Shetland Sheepdog (aka Sheltie – they are NOT even related to Collies, she is not a "little Lassie dog".)

Let these two numbers be x values, one on each side of the y-axis, that we substitute into our $y = x^2$ equation to get two points: $(10, 100)$ and $(-7, 49)$. Plot these points, connect them by a line, and find the equation of that line. Yes, you do know how to do this, it just might be very rusty. I will help you remember, just like Socrates.

Recall that the most commonly used equation of a line is the "slope-intercept" form: $y = mx + b$. Where m is the slope (the ratio of the change in "rise" to the change in "run"), and b is the y- intercept (where the line crossed the y-axis). To find the equation of a line you need either two points on the line, or one point and the slope. We have the points $(-7, 49)$, $(10, 100)$ and find the slope as follows.

$$m = \frac{\Delta y}{\Delta x} = \frac{100 - 49}{10 - (-7)} = \frac{51}{17} = 3$$

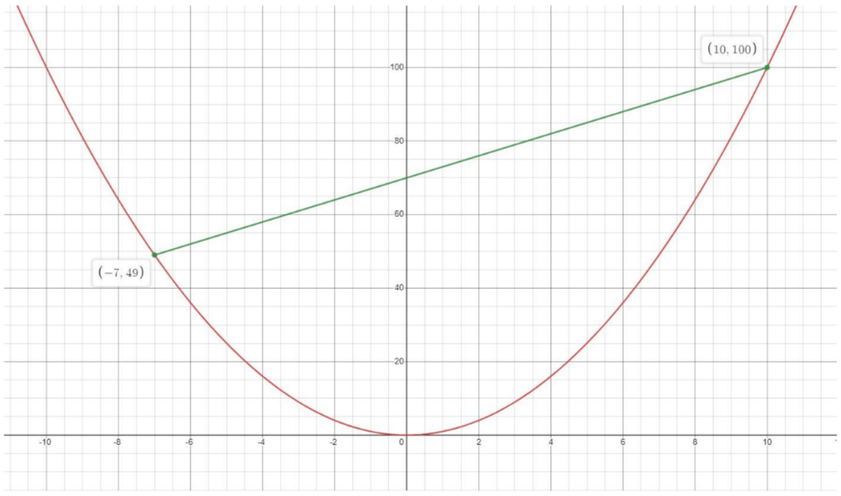
Which gives us $y = 3x + b$.

Now using either of our points (I like $(10, 100)$ for simplicity's sake) we get:

$$100 = 3(10) + b$$

Giving $b = 70$. Thus, the equation of the line connecting our two points is $y = 3x + 70$.

So where is the product of 10 and 7 in this equation/graph? It is the y-**intercept!**



Maybe this is a fluke. Try another pair of numbers, whole numbers, fractions, real numbers. As usual, I'll do some chores while you work on that.

...

Did it work? I hope so! But as any good mathematician would say, examples do not a proof make. We need to prove it for ANY and ALL numbers. That is the Algebra part.

Proof. Let A and B be any two positive numbers. We will let $x_1 = A$ be one input and $x_2 = B$ (with B plotted on the negative side of the x-axis) be the other. We want to show that AB is the same as the y-intercept of the line connecting the two points with y-values obtained from the parabola $y = x^2$. This gives us the ordered pairs (A, A^2) and $(-B, B^2)$. As above, we find the equation of the line connecting these two points. You will need to remember a little factoring, in particular my all-time favorite factoring special case, *Difference of Two Squares*.

$$m = \frac{\Delta y}{\Delta x} = \frac{A^2 - B^2}{A - (-B)} = \frac{A^2 - B^2}{A + B}$$
$$= \frac{(A+B)(A-B)}{A+B} = A - B$$

So now we have $y = (A - B)x + b$.

Using either point (I'll use (A, A^2) since my name is Amy and not Bob), we get:

$$A^2 = (A - B)A + b$$
$$A^2 = A^2 - AB + b$$
$$b = AB$$

And VOILA! The y-intercept is the product of the two x values, the numbers we wanted to multiply. Again, unless you are one of those people who like to do things the really hard way, don't multiply this way. But it sure is a cool connection between arithmetic, algebra, and graphing.

In closing, the first time I visited MoMath (The National Museum of Mathematics) in New York City, I was excited to see that the circular stairwell encases a large art installation of this property titled *String Product*. The outer structure is the parabola rotated to make a 3-D cage. Inside, colored "strings" connect whole numbers which show their product on the central vertical axis on glowing white tabs.

LEARN YOUR WAY

New to the PMP family in 2023, Michael Maser is an award-winning educator, author and learning coach who teaches Neurobiology and Learning in the Individual Masters program with Antioch University. He has just wrapped up his PhD, focused on learning, at Simon Fraser University, BC, Canada.

His website is www.michaelmaser.net.

Michael Maser

Hello and welcome to the second instalment of "Learn Your Way."

In my previous column I introduced the concept of 'learning detective' as a metaphor to support you in learning math or other subjects of interest.

I also left you with some homework, to identify the things you are pretty good at and your general or specific learning goals. Being honest and clear about these things will help you be more focused and successful.

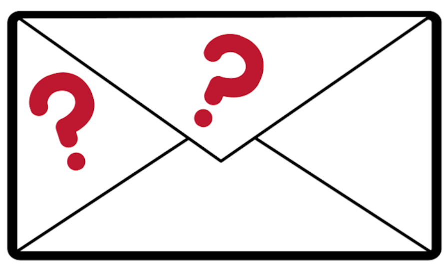
In this column I'm going to apply the 'learning detective' concept in studying another person to see how they achieved learning breakthroughs in the face of serious challenges, maybe similar to what you are facing or have endured. Our subject today is Christopher Havens, who is responsible for starting the PMP and the newsletter you are reading.

In a nutshell, Christopher is serving a 25-year sentence, at the beginning of which he found himself in solitary staring at the walls. It was a grinding environment, but Christopher turned it into his personal 'learnscape,' as he shared with me in a recent interview.

The details of his transformation are worthy of carefully unpacking to reveal the strategies he invoked. This is the stuff of learning detective work – noticing the *what* and *how* of somebody doing something you consider important. I urge you to try this because "success leaves clues," and these clues can also help guide you to learning success. Below is my summary of Christopher's story in which I unpack the specific things he did that ultimately helped him.

Strategies are real processes

1. While in 'the hole' Christopher noticed a staff member slipping envelopes to other inmates once a week. He asked what was going on and was told they were math problems that helped inmates pass the time. He was offered an envelope and he said yes. That is, he OPENED himself to be CURIOUS, to check this out for himself. Now, Christopher was not a math whiz; as a kid, he had average success with math but dropped out before finishing high school. So it took some COURAGE to take this first step and try something new.



2. Christopher liked trying to solve the problems and puzzles. The math was pretty basic but it made sense to him and he KEPT AT IT. In deepen-

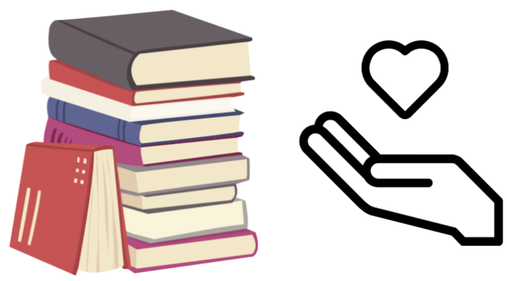
ing his FOCUS on the problems he began to experience real satisfaction and powerful sensations—like an ability to filter out the noise that was the soundtrack of life in the hole. And those walls he was staring at? With a little IMAGINATION he turned them into 'blackboards' on which he sketched out math equations and also taped real sheets of paper.

3. As he continued to experience success he began to recognize foundational truths in math that had been lacking in the experiences that had led to so many problems in his life and, ultimately, to prison. In another step of OPENNESS, he reflected on his own life and found himself facing a critical, life-changing decision. Again, he summoned the COURAGE to embrace change and continue deepening his commitment to math as a LIFE-CHOICE that helped him move away from the unstable world that had created problems for him.

"I began to think that if I really worked at it maybe I could become a 'mathematician,' " he said. "That was a huge breakthrough for me."

4. After making this choice, Christopher was willing to transform but it required a whole new level of COURAGE for him to move into unknown territory and basically CREATE A NEW REALITY for himself, including a new identity.

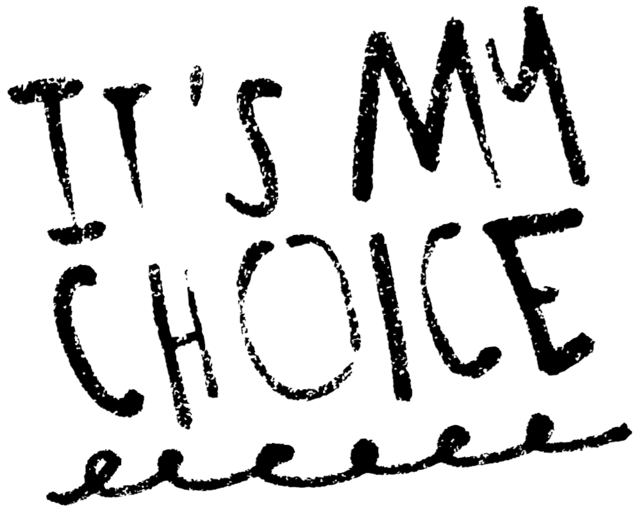
"At this point I had to restructure my values and my life," he said. To accomplish this he needed support and allies to help him stay on the path that was bringing him success. Released from the hole into the general prison population, Christopher bumped into inmates who expected him to act as his former self, leading to more stress. This is a hinge point that trips up many people trying to make important life changes, and Christopher was no exception. At one point he pleaded with staff to return him to the hole so he could avoid conflict. Staff initially dismissed him but he was eventually supported in entering a prison-based transition program. Two additional allies helped him at this time, including his mother who offered unwavering support, listening to him and fulfilling his requests for new math texts. A little later Christopher also gained a mentor beyond the prison, a professional mathematician in Italy willing to accept him as a long-distance student.



5. Christopher COMMITTED TO TRANSITIONING INTO A NEW LIFE, characterized by NEW IDEALS he had created. In math he continued to experience joy and satisfaction and it filled him up. But he also SAW A BIGGER PURPOSE here, too, and strove to create the PMP to HELP SERVE fellow inmates who also might engage with math, or something else, as a way to discover a new life of meaning and purpose around their passion for higher education.

"I didn't know if I was ready to do this," he said. "But I've learned that even inside a prison, the world can be cast in beauty and meaning. The PMP is an embodiment of this."

And that's where our learning detective work sums up for now. If you 'follow the puck' here you can see Christopher's transition was no accident but the result of deliberate choices he made. He faced real psychological challenges, candidly reflected on his life, saw a bigger picture of possibility and set new goals to achieve. His goals did not all appear at once, or even very clearly, but were more like stars twinkling through a cloudy night.



It's up to you to determine if or how the strategies Christopher invoked might apply to you. You must make your own choices for your own learning journey. And of course nothing will change if you don't set an intention to change. But once you set an intention in motion, the psychic force of this is powerful, as Christopher's story reveals. That helped him find the courage to face his past, keep trying math problems, and ask for help.

Your homework:

Observe somebody doing something you are curious about or admire, carefully noting the processes they seem to be following. Now, approach them and respectfully ask them how they are doing this and if they might help you learn it. I know asking for help is often challenging—especially in prison where it may be interpreted as a sign of weakness. I encourage you to work through this in your mind and also to practice it, aloud. If someone blows you off, roll with it and move on to ask someone else. You *will* find someone to respond to your request and show you what they're doing. Like most things, this will be easier the more you do it. Remember to thank them.

In my next column I will discuss the power of 'self-talk' to create your own blueprint for learning success.


Christopher Havens

A MISCREANT'S MISCELLANY

by Christopher Havens

Inclusivity and Human Flourishing

I believe that, in a way, **living** mathematics can be thought of as **doing** mathematics. This is especially true in the sense that contributing to the math community is also contributing to mathematics. Why? Because it creates an inviting version of our mathematical landscape for people you wouldn't normally expect to be **doing** mathematics—people who nonetheless contribute to the wealth of human knowledge, or people who further enrich our community in some other way.

My own mathematical journey began—literally—in isolation, a place most prisoners refer to as "the hole." My use of the word "*prisoner*" is a personal preference, by no means politically correct (unlike, say, "*justice-involved individual*"), but it is meaningful to me for two reasons:

Firstly, by definition, because I live in a prison, I am a prisoner. A quick glance in the dictionary confirms this:

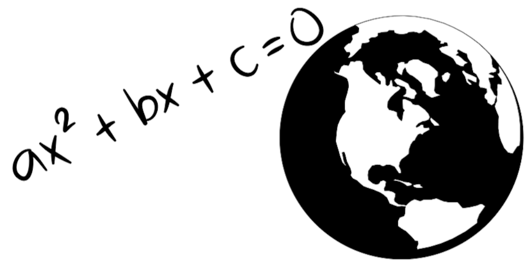
Prisoner: *1) A person deprived of liberty*
2) One on trial or in a prison

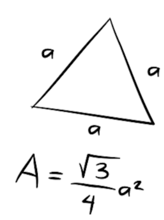
Although I no longer identify with the first part of that definition, it helped me to appreciate something I had taken for granted:

Liberty: *The quality or state of being free; FREEDOM.*

Very shortly after the start of my mathematical journey, a fire was lit inside me. I found myself undergoing an incredible personal transformation when I discovered beauty and joy through mathematics. This was also where I found true freedom.

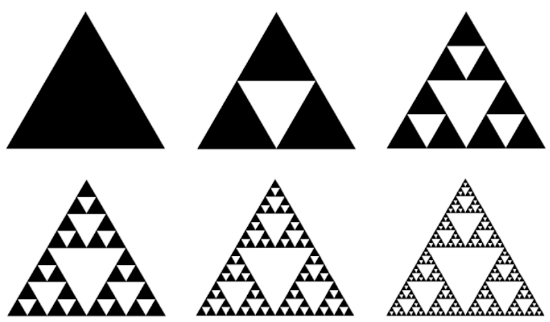
About two years after my very first steps along this numerical pathway, I found a mentor, an Italian mathematician named Luisella, who nurtured this intense fire. Our correspondences took place like the PMP mentorships do now. I'd send her letters across the ocean to her university in Turin and she'd write back. Not long after she introduced me to number theory, I began doing research with a group of Italian mathematicians. By 2018, my world was bathed in beauty, like a brilliant black and white masterpiece. I was in awe of this world and having some of the best times of my life, despite my being a prisoner. Strange, huh? But there's more. My beautiful black and white mathematical world was about to change.





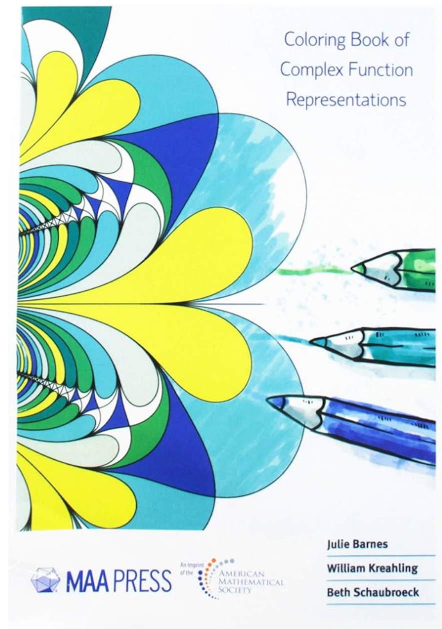
I didn't know it yet, but in 2018, while I was engaging in research and a meaningful mentorship, something big happened in the mathematical world. A mathematician named Francis Su gave his parting speech as the President of the Mathematical Association of America (MAA). This speech touched on the idea of mathematics as a tool for human flourishing and called out to the world for a more inclusive math community. Until then, this was a concept I had been completely oblivious to.

I was at the time involved in a project which led me to reach out to the MAA. In an attempt to find historical pieces of "Sierpinski art," I navigated my way into the History of Mathematics SIGMAA forum. To my delight, my favorite math historian, Dr. Amy Shell-Gellasch, responded. I was honored when our conversations led to my first mathematical writing project about the Sierpinski Triangle. I knew that this piece would require me to research the topic at a level beyond my actual resources, but I had this incredible yearning deep in my heart to write a quality article despite my limitations: no internet, no actual library, and none of the resources of a university. Under the circumstances, the only solution I could think of was to begin reaching out into the mathematical community and contacting each and every living mathematician and researcher who had contributed to the particular story of the Sierpinski triangle (ST) that I hoped to tell.



I read every piece of literature I could find on the ST. Eventually I had accumulated close to 100 journal articles—no joke. The topics ranged from Julia sets, cellular automata and the chaos game to 3D printing. From articles on medieval architecture to proofs without words, iterated function systems and on to the Tower of Hanoi—the list goes on and on. Needless to say, I explored every single one of these topics, and when I craved more about the creative moments and the motivation behind each facet of the story of the Sierpinski triangle, well, that's when I took my first blind steps into the human side of the world of mathematics.

The first person I reached out to was Tom Bates, a mathematician and artist who makes art using his own algorithms based on the chaos game. I remember how willing he was to help me tell my story, answering me so patiently while I bombarded him with questions both about the mathematics involved and his personal creative process.



I reached out to Beth Schaubroeck and Julie Barnes, two of the authors of the *Coloring Book of Complex Function Representations*, and many other mathematicians. When I introduced myself, I was often asked if I was *that* Christopher.

"*That* Christopher?" I would ask, puzzled.

"You know, the Christopher who studies math in prison."

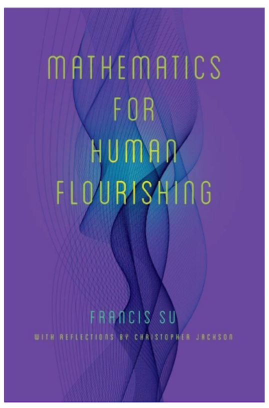
"I'm not sure whether I'm **that** Christopher, but I do know that I'm **this** Christopher, and I happen to study math in prison."

Well, that did the trick. Here I thought my efforts would be met with an enormous amount of skepticism on an already bumpy road. To be honest, I was expecting everyone to dismiss me. I didn't know yet that the math community is much more supportive than I could have imagined.

By the time I had reached out to a dozen mathematicians, at least three had made comments about some **other** Christopher, and they all referenced a speech by someone named Francis Su. It always excites me to hear about another prisoner whose actions shout into the world to the point where the world truly sees them as somebody whose newfound path has given form to the true meaning of justice. More often than not, the media showcase the world's most dangerous and violent prisoners, which leads the nation to believe that all prisoners are of that variety. If society had a more accurate view of prisons and prisoners, perhaps those members of society who enter prison for the first time wouldn't feel they had to use violence as a defense mechanism. The culture of prisons would change, and our wonderful, anomalous pack of "Christophers" wouldn't be such an anomaly because *reform. might. be. more. common*. But I digress...

I was excited and I was intrigued, so I looked up Francis Su and found his speech—and finally it all made sense! There was another Christopher experiencing mathematics - I imagined - just like I was. Math is sacred to me because it saved me from a life of imprisonment, pain and chaos. It opened my eyes to see beauty in everything around me. It opened me to love.

Francis Su's speech sent prickles down my arms, because his words spoke directly to my heart. It was a clarion call for inclusivity in the mathematical community, and every time I reached out to another mathematician who wanted nothing more than to help unconditionally, I knew that I was experiencing the immediate effects of his speech, *Mathematics for Human Flourishing* (which became the basis for his book).



As I charted my new path, I embarked on elaborate journeys with mathematicians across the world. While my body was stuck in a small prison cell, I chased mathematical secrets down the side streets of Italian towns and networked my way into the Royal Society and beyond. These imaginary escapades capture what it *felt* like to peer so intimately into mathematical history and culture. Those pictures in my mind were made real in my heart through so many wonderful interactions with the math community—but I know my experience would have been different had the climate in the world of math not been set through the relationship between Christopher Jackson and Francis Su. My beautiful black and white world of mathematics was now full and vibrant colors I could not even have imagined.

The truth is that my road into the mathematical community was paved in bright yellow bricks, just like the one in the land of Oz. Unfortunately, not everyone gets to experience this, and if my enquiries into the math community had come a year later or a year earlier, all those wonderful memories would probably look quite different. I know one thing: my positive experience entering the community made me strive to be worthy of what I am so fortunate to have. My mathematical life really has been surreal, and now I try to project my ideal version of the math community back into the world so that others can feel the joy that I feel.

I believe Francis does this too.

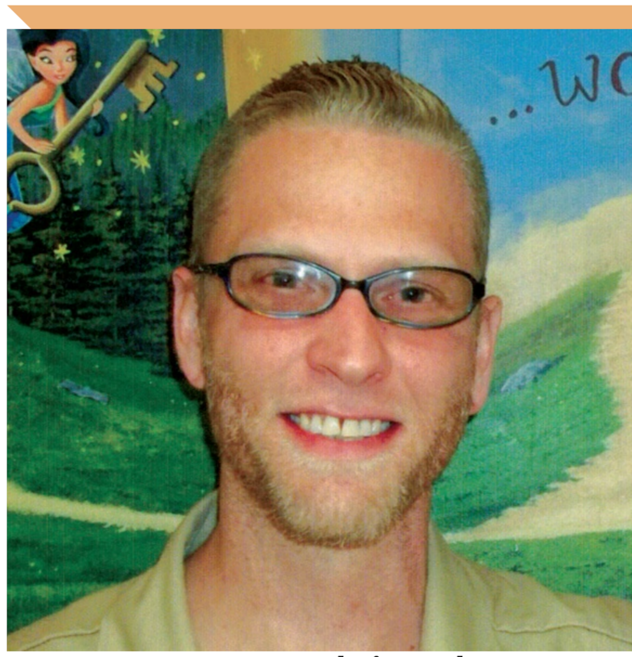
We should all try to put forth our best version of "mathematics," so that the next person can flourish and grow within the ripples of our ideal community.

Echoing the words of my friend, PMP fellow Paul Morton, who is introduced to you in these pages, I don't want to mislead anyone into thinking the world of mathematics is made up of unicorns and stardust. But when the community works towards inclusivity, it affects people's lives—like it did mine. From that single push toward inclusivity, I've been able to build bridges that have helped so many people. This is how I serve justice.

In closing I would ask that each one of you project into the world only that version of the community that your heart truly yearns for. You never know: it may just be the opening somebody needs to truly succeed in finding their place in the world.

—"*This*" Christopher

CLOSING COMMENTS



Christopher Havens

Greetings friends!

As we close this iteration of the Prison Mathematics Project newsletter, there are a few things I'd like to put out there. We really do strive to make our newsletter enjoyable for our incarcerated participants, as well as those in the broader math community. So whether you're sitting in a prison or the university faculty lounge, we hope that you can jump in and connect, however that may look. Whether you connect through one of our wonderful columns, contributing one of your own, or even just posting our “invitation” on page 2 where others will see it, we believe that through an inclusive community, mathematics can change lives and help the process of rehabilitation among our nation's incarcerated population.

Please remember that in order for PMP to help you stay connected to mathematics after release, we request that you communicate your release date and any other relevant information so we can help the process of your re-entry.

For educators, graduate students and members of the math community: If you feel like creating prison-specific mathematics material, we would like your help tailoring unaccredited course material for our participants in all areas of math. For more information, please send a message to christopher@pmathp.org.

Christopher

PMP

PRISON MATHEMATICS PROJECT

10810 N. Tatum Blvd.
Suite 102-998
Phoenix, AZ 85028