PMP

PRISON MATHEMATICS PROJECT

THE PRISON MATHEMATICS PROJECT NEWSLETTER

SPRING 2023 – ITERATION 6

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Welcome to our latest iteration of the Prison Mathematics Project newsletter. There's something in these pages for everyone—from the merely math-curious to those pondering the most intriguing and complex mysteries of the universe.

Our writers, editors and contributors from around the globe have designed this newsletter to share with you our deep appreciation for the wonders waiting to be discovered through the study and exploration of mathematics. We connect prisoners who are dedicated to change with a community of people who share their passion for mathematics. In this way we provide an essential framework for rebuilding their lives during their incarceration.

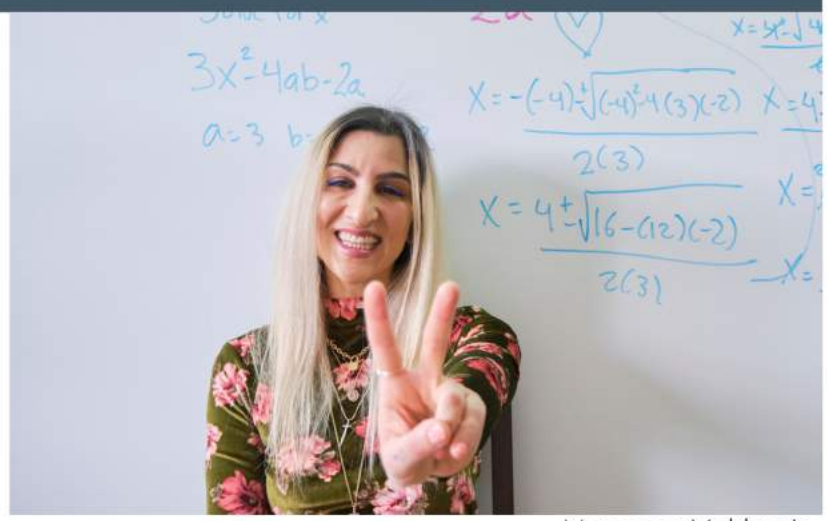
Together we enter a world that transcends walls, where all are welcome and anyone who wants to can embrace the beauty, the humanity and the transformative power of mathematics.

You can become an active participant in your own rehabilitation. We invite you to experience a culture that leads to desistance from crime and meaningful employment opportunities post-incarceration, but more than that, to human flourishing. Wherever you are on your own journey into the realm of mathematics, we're here for you.

ASK THE MATH GURU

Do numbers freak you out? Does math send you into meltdown-mode? Are you wondering why we need to learn this stuff ANYways?? Don't worry, The Math Guru is here to help you work through your math trauma, one problem at a time. Ask for advice, guidance, or just a good ol' pep talk! You got this!

☮♥π



Vanessa Vakharia,
The Math Guru

On March 14 2023, I had the privilege of hosting PMP's first-ever LIVE Pi Day event at Bellamy Creek Correctional Facility in Michigan. To say that the event was AMAZING would be an understatement! Seeing PMP come to life in the Bellamy gymnasium was awe-inspiring and humbling, and truly demonstrated the power math has to heal, empower and equalize.

For this edition of the newsletter, I wanted to share one of the questions I received at Bellamy because I know that a LOT of you have this same question!

Dear Vanessa,

What is the best way to go from regular math to algebra if you don't have much background knowledge in math?

Nathan Wilson

I am SO glad you asked this question, Nathan - it's a question that many of you have asked! A lot of things in math are scaffolded, which means you need to understand one concept to get to the next concept. BUT a lot of topics are NOT scaffolded - and this is important to know. Just because you don't understand one thing in math doesn't mean that you can't understand ANYTHING in math. Remember that there are many branches of mathematics, so keep exploring to see what gets you excited!

Now, onto the actual question: I would recommend getting a handle on fractions, BEDMAS (which is just order of operations) and integers before you head on over to algebra. That should give you a nice foundation before you start bringing the alphabet into math. Everyone thinks that mixing letters in with numbers is when math gets scary, but from witnessing all of you at Bellamy - so many of you were doing algebra without even knowing it! In math, letters are just placeholders for unknown numbers. So for example, you might say something like x+3 = 5 and then ask what is x? So think about it right now! SOMETHING plus 3 adds up to 5. What do you think the "something" is? Chances are that even if you don't know how to technically solve the equation, you might still intuitively know that the answer is 2. If you did...YAY! You just did algebra! HIGH FIVE!

Hopefully that helps! Thank you for asking the question, and remember that math is ALL about following your curiosity to see where it leads you - the journey is more important than the destination :)

☮♥π

Vanessa Vakharia

Got math anxiety? Think you're a hopeless case? VaneSSa to the rescue! The Lady Gaga of mathematics will put her Master's in Mathematics Education to good use by delivering the ultimate personalized pep-talk!

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MATH THERAPY

NOW AVAILABLE ON SECURUS

Vanessa Vakharia (our very own Ask the Math Guru columnist) hosts a weekly podcast called Math Therapy!

Math Therapy is the inspirational podcast that works through your math trauma, one problem at a time! Each week, Vanessa Vakharia (aka The Math Guru) chats with a different guest about their relationship with math and explores the educational highs and lows that can make or break our self-confidence. Whether you think you're a "math person" or not, you're about to find out that math people don't ACTUALLY exist - but the scars that math class left on many of us DEFINITELY do! And don't worry - no calculators or actual math were used in the making of the podcast ;)

To listen to Math Therapy

1) Open the podcast app on your Securus device.

2) Select 'Add Podcast' from the dropdown menu.

3) Click "Search" and enter the name 'Math Therapy'.

4) Enjoy!

THE PRISONER'S DILEMMA

From the Problem Warden

I am delighted to be part of the Prison Mathematics Project through The Prisoner's Dilemma, *as its (honorary) "Problem Warden." My love of mathematics – and especially sharing the joy it can bring – have been a part of almost everything I've done as a professor, financial analyst, parent, founder of the National Museum of Mathematics, author of the Studio Infinity blog, and the most recent ex-editor of* The Playground *(the problem column of* Math Horizons, *a magazine chronicling the world of math for the Mathematical Association of America). I look forward to seeing all of the new methods and creative questions that you come up with as we face the many dilemmas to come, together. And don't worry, as a one-time specialist in mathematical logic, this Warden will keep a sharp eye out for any infractions in your reasoning!* **– Glen Whitney**

Glen Whitney, AKA The Problem Warden

D10: Half-and-half Hanoi

Contributed by Andreas Hinz, LMU München

You may be familiar with the Tower of Hanoi puzzle in which you start with eight discs stacked on one of three pegs, each disc smaller than the one below it. The other two pegs are empty. The goal of the puzzle is to move the entire stack of discs to a different peg, according to the following rules:

- a) Only one disc may be moved at a time, from the top position on one peg to the top position of a different peg.
- b) No disc may ever be placed on top of a smaller one.

For convenience, we number the discs from 1 to 8 in order from smallest to largest. In this variation, you start with discs 1, 3, 5, and 7 on the first peg, and discs 2, 4, 6, and 8 on the second peg, as shown in **Figure 1**. Your goal is to end up with all eight discs in order on the third peg, following both rules (a) and (b). What's the fewest number of moves needed to accomplish this goal, and how do you do it? How do you know your solution is minimal?

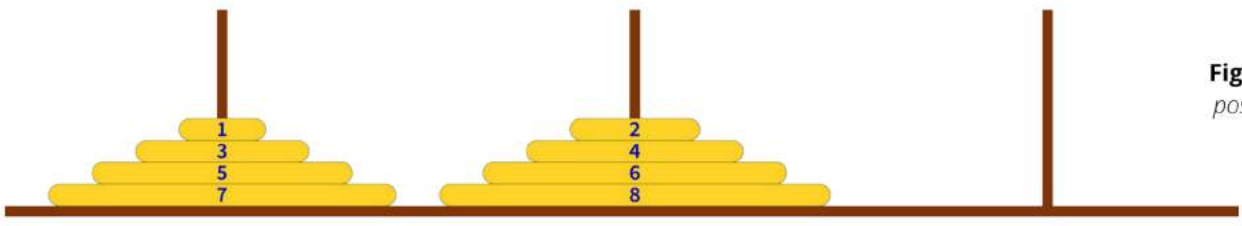


Figure 1: *The starting position for Half-and-half Hanoi.*

A dilemma of a different sort has come to our attention: sometimes it's difficult for these problems to get to you, and for you to have time to think about them and try your hand at solving them, and then for your solutions to get back to the Problem Warden. So with this issue, we're instituting a revised schedule for the new problems: **there will be twice as long for your submissions to make their way back to us**, and so solutions will be printed two issues from now, not next issue. But don't worry, we'll still have new Dilemmas to bend your brain, in every iteration. In fact, because of the extra time, we're increasing the number of new problems to four. Hopefully that way for every reader, there's at least one that will tickle some interest. This change also allows us to feature a record-breaking four different contributors, from around the world and from both inside and outside of the PMP.

D11: Octosection

Contributed by Arsalan Wares, Valdosta State University

It's high time for another one of Prof. Wares' beautiful geometric Dilemmas. For the setup, refer to **Figure 2**. Square *ABCD* has been dissected into eight equal-area rectangles, as shown. The width of the rectangle shaded red in Figure 1 is 35 units. What is the area of square *ABCD*? For a bonus challenge, can you devise and solve a similar problem that dissects a square into eleven pieces?

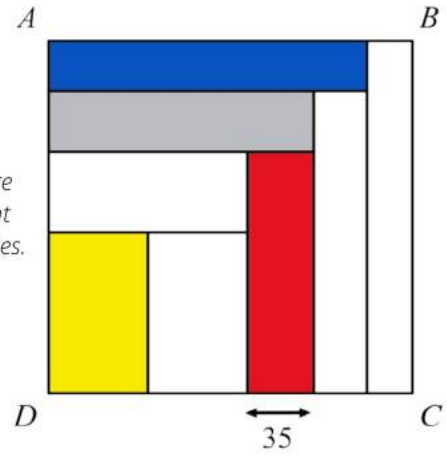


Figure 2: *A square dissected into eight equal-area rectangles.*

D12: Strange Coincidence

Contributed by Ian Stewart, University of Warwick

Speaking of returning contributors, Ian Stewart is back with another one of his riddles. This time, it's about a family divided by math, so to speak. Let's listen in on their conversation:

"That's amazing!" said Mathophila. "The squares of three consecutive positive whole numbers add up to the same number as the squares of the next two numbers."

"Oh, no big deal, that must happen all the time," replied her brother, Innumeratus.

"I don't know, it seems like an amazing coincidence to me!" retorted Mathophila.

Find the smallest collection of numbers with the described property, and settle the argument between the siblings: determine whether there are infinitely many sets of numbers as described by Mathophila, or a finite number (and if so, figure out exactly how many solutions there are).

D13: The Experbola

Contributed by William Keehn, PMP participant

Last iteration we had a problem (see D8 below) investigating a curve similar to an ellipse, but changing addition to multiplication in its definition. This time, PMP participant William Keehn proposes a similar exploration related to the hyperbola, another conic section, which can be thought of as all of the points (x, y) in the plane where $xy = 1$. Since that description already involves multiplication, this time we will move on to exponentiation.

So, now consider all of the points where $x^y = y^x$ such that x and y are *distinct* positive real numbers. You get a curve similar to the hyperbola, that William has dubbed the "experbola." **Figure 3** shows the hyperbola and experbola side-by-side. And William is brimming with interesting questions about the experbola, so Dilemma 13 has lots of parts (feel free to submit answers for any or all of them):

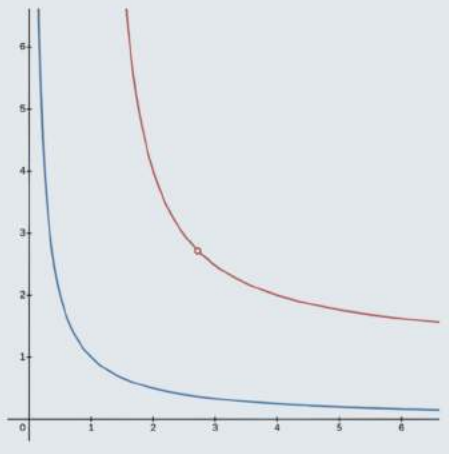


Figure 3: *A hyperbola (in blue) and the experbola (in red). Courtesy of desmos.com*

a) Find all of the points on the experbola where both coordinates are whole numbers. (And explain why there aren't any others.)

b) An *asymptote* of a curve is a straight line that the curve becomes arbitrarily close to but never touches. For example, the hyperbola in Figure 3 has two asymptotes: the lines $x = 0$ and $y = 0$. Does the experbola have any asymptotes? If so, find them.

c) The condition that x and y are distinct is needed because whenever they are the same, the equation $x^y = y^x$ is satisfied. So without that condition, the experbola would include the entire diagonal line $x = y$. But as you can see in Figure 3, that condition also cuts the experbola into two disconnected pieces. However, there is a unique single point on the line $x = y$ that you could add back to the experbola to "complete" it and make it a single continuous curve. (That's the point indicated by the small red open circle in Figure 3.) What are the coordinates of that point?

d) Find (non-constant) functions $f(t)$ and $g(t)$ such that the point $(f(t), g(t))$ is on the experbola for all positive real numbers t. For this one, William offers a hint, by way of an example: $\sqrt{3}^{3\sqrt{3}} = (3\sqrt{3})^{\sqrt{3}}$. Such a pair of functions is called a *parametrization* of the experbola, or at least of the part of the experbola it lands on.

D4: Four Operations Revisited

(First appeared in Iteration 3, Winter 2022.)

This problem has been long-standing, seeing submissions over several issues. There is just the multiplication part left, restated here as a stand-alone problem. This is the only problem we'll print the solution for in the next issue due to the new schedule, so if you can submit a possible solution by **2023 July 15**, we'll include your work then. (As mentioned in the submission guidelines below, solutions for all other problems are due 2023 Nov 1.)

Consider the sequence m_1, m_2, m_3, ... defined by $m_1=1$, $m_2=2$, and $m_{n+2} = m_{n+1}m_n/2 + m_{n+1}$, which can be rewritten $m_{n+2} = m_{n+1}(m_n + 2)/2$. Can you find closed-form sequences (given by formulas in terms of n that don't involve earlier terms) l_n and u_n , and a constant N, such that for all $n > N$, $l_n < m_n < u_n$? The closer together your l_n and u_n are, and the smaller N is, the "better" your answer would be considered.

SECOND HEARING

Here's the portion of the column in which we revisit prior dilemmas, to share solutions that readers have come up with, or report on their status.

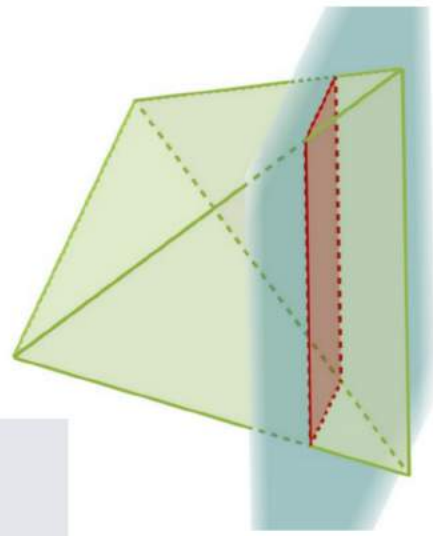


Figure 4: *How to get rectangular cross sections of a tetrahedron*

D7: Three and a Half Sides

Inspired by Jim Propp, University of Massachusetts, Lowell

If you orient a tetrahedron so that both of a pair of opposite edges are parallel to a fixed "cutting" plane and then pass it through the plane, all of the cross sections are rectangles, as shown in **Figure 4**. Can you find an orientation of the tetrahedron so that when you move the tetrahedron at a constant rate perpendicular to the cutting plane, the *average* number of sides of the cross-sections formed is three and a half?

Prof. Propp also proved that no matter how you orient a cube, when you pass it through a cutting plane in this way the average number of sides of a cross section is always exactly four. A regular octahedron is more like a tetrahedron: if you slice it perpendicular to a line joining two opposite vertices, all of the cross sections are squares, but if you slice it parallel to one of the faces, all of the cross sections (except the lone initial and final ones) are hexagons. What about the other two Platonic solids, the dodecahedron with twelve pentagonal faces and the icosahedron made up of twenty equilateral triangles?

Solution to D7:

We received a solution from William Keehn and a partial solution from Jesse Waite of Fort Leavenworth. Both submitters realized that one straightforward way to control the numbers of sides of the cross-sections is to tip back the top-right vertex of the tetrahedron in Figure 4, straight toward its leftmost horizontal edge. This tipping is equivalent to rotating the tetrahedron around that leftmost edge (before starting to slide it through the cutting plane). When you do this, the cross sections have three sides until that top-right vertex reaches the cutting plane, and four sides thereafter.

Finishing the problem off just amounts to computing the angle at which to tilt the tetrahedron so the top-right vertex reaches the cutting plane exactly halfway along the passage of the tetrahedron through the cutting plane. Then the average number of sides of the cross-section will be 3.5, as desired. To do this, we reduce the three-dimensional problem to one in the plane, by looking directly along the rotation axis (the left horizontal edge). Note the projection of the tetrahedron along this direction is not an equilateral triangle, but rather an isosceles triangle: all four edges proceeding from left to right in **Figure 4** are foreshortened in this view. The top two end up superimposed on the altitude of the top-left face, which maintains its length because it is perpendicular to the axis we are looking along. Thus, assuming the edge length of the tetrahedron is one, the foreshortened edges in the projection end up with a length of $\sqrt{3}/2$, the height of an equilateral triangle with unit sides. In other words, from the side, the tipped tetrahedron ends up looking like **Figure 5**, where T is the top-right vertex, R corresponds to the left-hand edge of the tetrahedron that has projected onto a single point (since we are looking directly along that axis), and B is the bottom-right vertex.

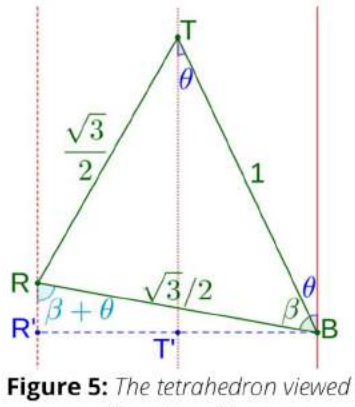


Figure 5: *The tetrahedron viewed from the side*

From this point of view, the cutting plane becomes a straight vertical line. Three different positions of the cutting plane relative to the tetrahedron are indicated in red in **Figure 5**: the initial contact at point *B*, the final exit at the line through *R* and *R'* , and exactly half-way at the line through *T* and *T'*. The blue dashed horizontal line is the line through *B* perpendicular to the cutting plane; you can think of the point *B* as traveling along this line or equivalently of the cutting plane as sliding along this line, staying perpendicular to it.

We want to solve for the tipping angle, represented by θ in **Figure 5**. The condition on the average number of faces corresponds to the requirement that *BT'* = ½ *BR'* . From the right triangles formed in the diagram, we have that $BT' = \sin\theta$ and $BR' = (\sqrt{3}/2)\sin(\beta+\theta)$. Substituting to eliminate *BR'* and *BT'*, we have $\sin\theta = \sqrt{3}\sin(\beta+\theta)/4$.

To finish, we need the sum formula for sine, namely $\sin(\beta+\theta) = \sin\beta\cos\theta + \cos\beta\sin\theta$. To take advantage of this formula, we need the values of $\sin\beta$ and $\cos\beta$. Fortunately, they are not too difficult to calculate because we know all of the sides of isosceles triangle *BTR*. Dropping a perpendicular from *R* to the midpoint of *BT*, we find that $\sin\beta = \sqrt{2}/3$ and $\cos\beta = 1/\sqrt{3}$. Putting all these ingredients together, $4\sin\theta = \sqrt{2}\cos\theta + \sin\theta$. Solving, we determine that $\theta = \arctan(\sqrt{2}/3) \approx 25.24°$.

As for the dodecahedron, Jesse noticed that if you pass it through a cutting plane perpendicular to the line joining two opposite vertices, all of the cross sections are either triangles or hexagons, so that average is less than six. On the other hand, if you pass it through a plane perpendicular to the line joining the centers of opposite faces, then the cross sections are pentagons except for a band around the middle where they are decagons. You can look up (or derive) a coordinate system for a regular dodecahedron where there are only four different *z*-coordinates for the vertices: $\pm\phi\pm1$, which means that the bottom face must be perpendicular to the *z*-axis. (Note that ϕ here means, as usual, the golden ratio $\phi = (1+\sqrt{5})/2$) Using these coordinates, the height of the entire dodecahedron is $2\phi + 2$ of which $2\phi - 2$ is the height of the portion where the cross sections are decagons (and so there are pentagon cross sections for the remaining 4 units of height). Thus, the average number of sides of a cross section in this direction is

$$\frac{10(2\phi-2)+5\cdot4}{2\phi+2} = \frac{20\phi}{2\phi+2} \approx 6.18 > 6.$$

Therefore, the average number of cross-section sides depends on the orientation, like it does with the tetrahedron and octahedron.

That's also the case with the icosahedron. Here we observe that the cross sections perpendicular to a line joining the midpoint of two opposite edges are all octagons (except for finitely many individual cross sections, which don't contribute to the average, since there are infinitely many octagonal cross sections). However, cutting perpendicular to the line joining two opposite vertices again results in pentagons and decagons. In this direction, the icosahedron conveniently decomposes into two pentagonal pyramids and a pentagonal antiprism. Again, one can look up the heights of these solids to find that the average number of sides in this direction is

$$S_v = \frac{10\sqrt{1+1/\sqrt{5}}+5\sqrt{1-1/\sqrt{5}}}{\sqrt{1+1/\sqrt{5}}+\sqrt{1-1/\sqrt{5}}}.$$

The first step in simplifying this formidable-looking expression is to clear the denominator by multiplying top and bottom by $\sqrt{1+1/\sqrt{5}} - \sqrt{1-1/\sqrt{5}}$. Following that trick with a bit of algebra will show that $S_v = 5\phi \approx 8.09 \neq 8$, so again the average depends on orientation. ■

D8: Mullipse?

Recall that given two points in the plane, an ellipse can be defined as the collection of points whose sum of distances to the given points is constant. But here in the Prisoner's Dilemma, we like to investigate other operations than addition, as in **D4** above. So suppose you are given points *F* and *G* a distance two units apart in the plane. We write just *XY* for the distance between points *X* and *Y* in the plane (so *FG* = 2, for example). Describe the collection of all points *M* in the plane such that *FM* · *MG* = 1, five of which are shown in **Figure 6**. You might also want to consider what the collection of points *P* such that *FP* · *PG* = *c* looks like for other values of *c*.

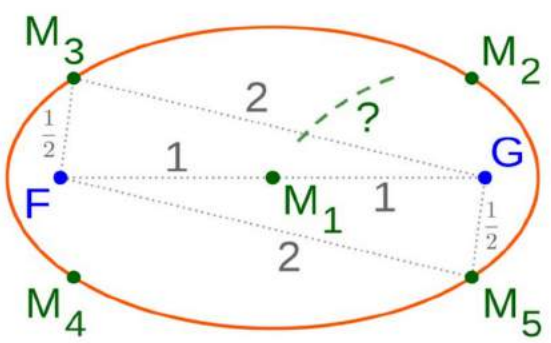


Figure 6: *Five points (in green) on a curve of "constant distance product." Where does the curve go between these points?*

Solution to D8:

PMP participants William Jones of FCI Loretto, William Keehn, and Jesse Waite submitted solutions to this problem. As Mr. Keehn pointed out, the derivation is easiest in polar coordinates; see **Figure 7** for an illustration of the coordinates (r, θ) of a general

point *M* on the mullipse.

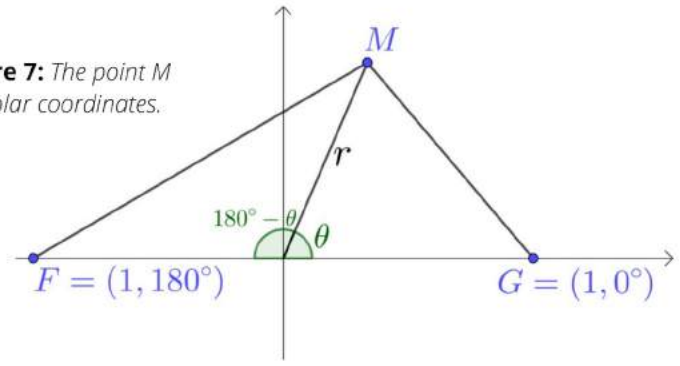


Figure 7: *The point M in polar coordinates.*

We have by the Law of Cosines that

$$MG^2 = 1^2 + r^2 - 2r\cos\theta.$$

Similarly, because $\cos(180^\circ - \theta) = -\cos\theta$,

$$FM^2 = 1^2 + r^2 + 2r\cos\theta.$$

The definition of the curve we want is that $FM \cdot MG = 1$, so we also have that $FM^2 \cdot MG^2 = 1$. Substituting and using the difference of squares, these observations mean that $1 = (1 + r^2)^2 - (2r\cos\theta)^2$. Finally, multiplying out, collecting terms, and using the cosine double-angle formula gives us:

$$1 = 1 + 2r^2 + r^4 - 4r^2\cos^2\theta$$
$$r^4 = 2r^2(2\cos^2\theta - 1)$$
$$r^2 = 2\cos 2\theta$$

This last equation gives us a pleasantly simple (polar) equation for the mullipse. You can see it as the green curve in **Figure 8**. (An equation for it in Cartesian coordinates is

$$4x^2 + 1 = (x^2 + y^2 + 1)^2,$$

which you can find by a similar process.)

As for what you get when the desired product 1 is replaced by some other constant *c*, you can re-do the same derivation starting from $FP^2 \cdot PG^2 = c^2$ to get the formula $c^2 - 1 = r^2(r^2 - 2\cos 2\theta)$. These curves for *c* equal to 0.5, 0.75, 1, 1.25, and 1.5 are shown in **Figure 8**. In case you'd like to look up more information about them, the mullipse is more widely known as the "lemniscate of Bernoulli" and the whole family of curves created by varying the constant *c* are called "Cassini ovals." ■

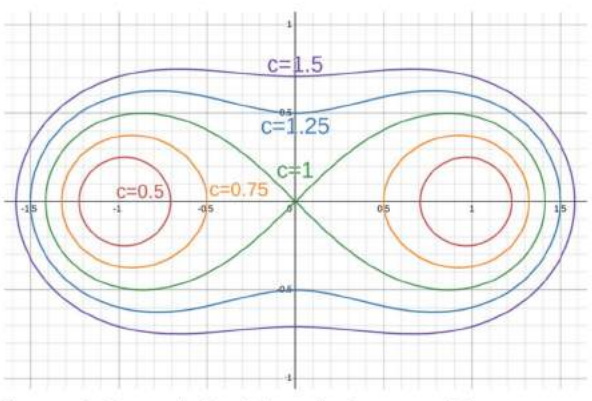


Figure 8: *Example Cassini ovals. Courtesy of desmos.com.*

D9: Semi Inellipse

Contributed by Paul Morton, PMP

PMP participant Paul Morton has also investigated the properties of ellipses. As shown in **Figure 9**, semiellipse *CTI* is inscribed in right triangle *CBA* so that it is tangent to hypotenuse *AB* at *T*, and so that points *I* and the two foci *F* and *G* of the ellipse cut leg *AC* of the triangle into four equal segments. What is the ratio of the length of hypotenuse *AB* to the length of leg *AC*, or in symbols, *AB* / *AC*?

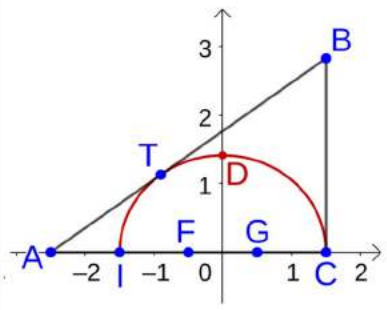


Figure 9: *Semiellipse inscribed in right triangle CBA so that its foci and other vertex cut a leg in four equal parts.*

Solution to D9:

This problem was solved by Jesse Waite and a collaborative team of William Jones and Ted Estes, also of FCI Loretto. William and Ted impressively used some techniques of calculus that William had just mastered. Here, we follow Jesse's alternate approach.

For convenience since segment *AC* is divided into four equal parts, we introduce coordinates scaled so that *AC* = 4. We place the origin at the center of the (semi)ellipse, i.e., at the midpoint between the foci *F* and *G*, so that the equation of the ellipse will have the standard form $x^2/a^2 + y^2/b^2 = 1$ for constants *a* and *b* giving the positive *x* and *y*-intercepts of the ellipse. In **Figure 9**, we've shown this coordinate system.

The coordinates of some of the points are immediate:

A (−5/2, 0), *I* (−3/2, 0), *F* (−1/2, 0), *G* (1/2, 0), and *C* (3/2, 0).

Since *C* is the *x*-intercept of the ellipse, we have *a* = 3/2. We have also added the *y*-intercept *D* (0, *b*) in **Figure 9**. Since *FC* + *CG* = 3, we have by the definition of an ellipse that also $3 = FD + DG = 2\sqrt{b^2 + 1/4}$, from which we solve for $b = \sqrt{2}$.

Now suppose the coordinates of point *B* are (3/2, *h*). Then the equation of line *AB* is

$$y = \frac{h}{4}\left(x + \frac{5}{2}\right).$$

Substituting this value of *y* into the equation for the ellipse to solve for the *x*-coordinate of point *T* (since the equations of both the ellipse and the line must be satisfied by point *T*), we get

$$\frac{4x^2}{9} + \frac{h^2}{32}\left(x + \frac{5}{2}\right)^2 = 1.$$

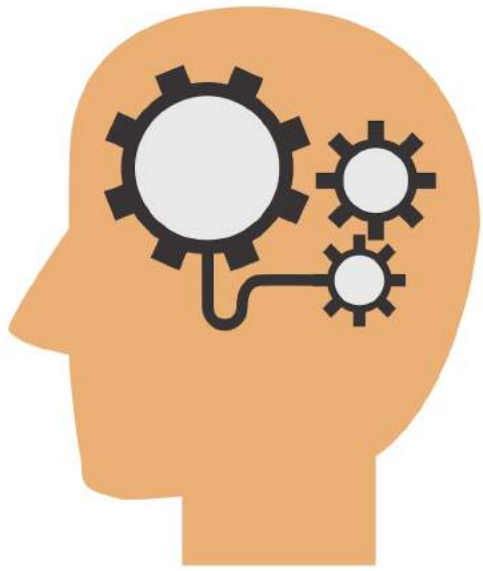
We can write out the square and multiply both sides by $9 \cdot 32 \cdot 4 = 1152$ to obtain the quadratic

$$(36h^2+512)x^2+180h^2x+(225h^2-1152) = 0.$$

However, it is given in the problem that the line *AB* is tangent to the semiellipse, so they only intersect at a single point. Therefore, this quadratic must have only one real root. But by the quadratic formula for $Ax^2 + Bx + C = 0$, namely $x = (-B \pm \sqrt{B^2 - 4AC})/2A$, there are two roots unless $B^2 - 4AC = 0$. In other words, it must be the case that

$$(180h^2)^2 = 4(36h^2+512)(225h^2-1152).$$

Again multiplying out, this equation simplifies to $h^2 = 8$, amazingly enough! Armed with this information, it is easy to compute that $AB = \sqrt{16 + h^2} = 2\sqrt{6}$, so $AB/AC = \sqrt{6}/2$. ■

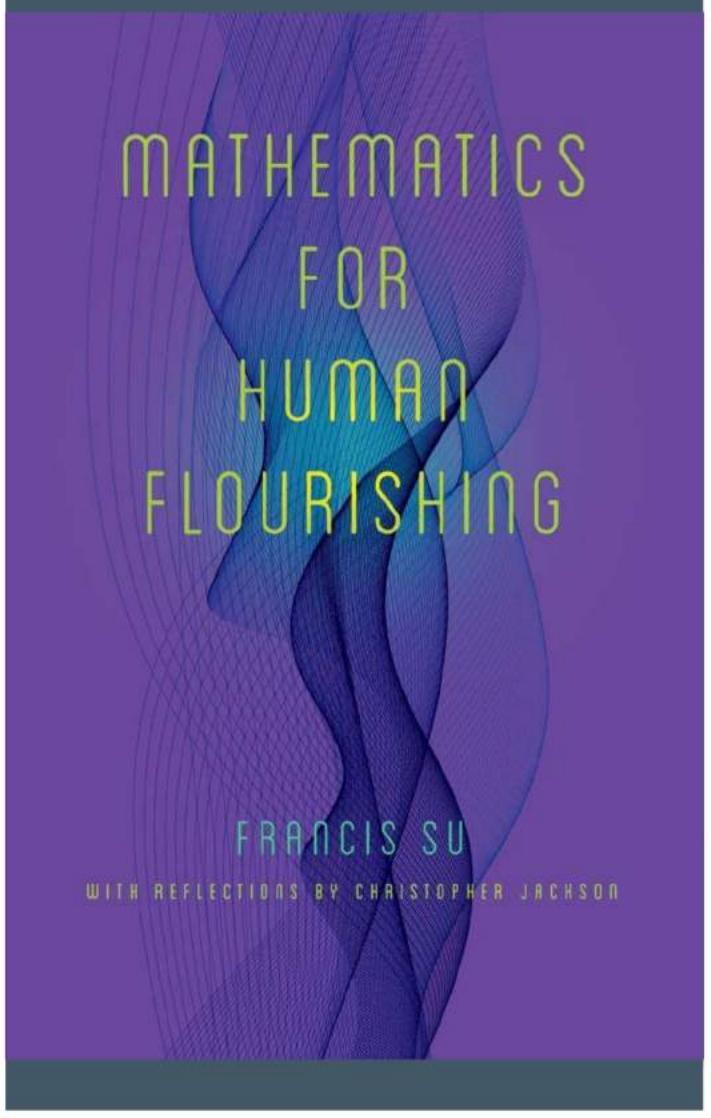


Submission Guidelines

Solutions to problems published in *The Prisoner's Dilemma*, and proposals for new Dilemmas, are welcome. For solutions, please clearly indicate the Dilemma number being solved. If a problem has multiple parts, you may submit solutions to any individual part or parts. Solutions to the Dilemmas in this newsletter must be received by the deadline of **2023 Nov 1** (except as noted for D4, which is already on its Second Hearing), and they will appear in the iteration *after* the next one. Dilemma proposals will be considered on an ongoing basis. All submissions should be addressed to Glen Whitney/Prisoner's Dilemma either by email at dilemma@pmathp.org (in which case PDF format is preferred, if possible, although any reasonable format will be accepted), or by mail at:

Prison Mathematics Project
Glen Whitney/Prisoner's Dilemma
10810 N. Tatum Blvd, Suite 102-998
Phoenix, AZ 85028

BOOK REVIEW CONTEST



ANNOUNCING OUR FIRST READING RECOMMENDATION FOR 2023:

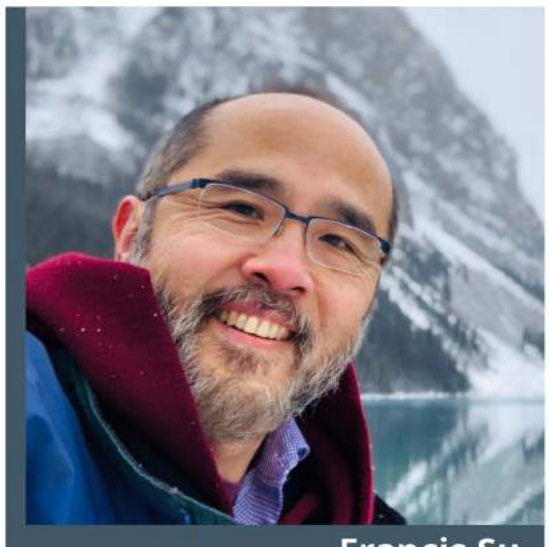
MATHEMATICS FOR HUMAN FLOURISHING
by Francis Su

"Perhaps the most important mathematics book of our time."

-James Tanton, *Global Math Project*

"A lyrical meditation on the beauty of mathematics and how it connects to our common humanity."

-John Urschel, author of *Mind and Matter*



Francis Su

Mathematics for Human Flourishing
(Yale University Press, 2020)
by Francis Su, with reflections by Christopher Jackson.

At its core, "Mathematics for Human Flourishing" is a meditation about what it means to be human. Su explains how the study of mathematics can meet many of our natural human desires and build virtues that help us flourish, even in the most difficult circumstances. Su's incarcerated friend Christopher Jackson, who discovered his own love of math in prison, exemplifies this truth. Through the arc of the book, in Jackson's letters and Su's narrative, we see how our humanity reveals that everyone is a 'math person.' For this effort, Su and Jackson were awarded the 2021 Euler Book Prize from the Mathematical Association of America.

ITERATION 6

Our last featured text was "*Explaining Logarithms: A progression of ideas illuminating an important mathematical concept*," by Dan Umbarger, and our review winner was Paul Morton in New York. Congratulations and thanks for your submission, Paul!

Here's how the review contest works:
You read the featured book and submit a review (see details below). If your submission is deemed most inspiring to potential readers by our panel of judges, it will be published in a future edition of the newsletter and on the PMP website.

One of our participants, James Conway, has already read the book and written his review. We think it's good enough to act as a "teaser." Your review doesn't have to be this long—or this literary—just let us know what moved you and why. We hope this advance review will whet your appetite and reviews will pour in so he has some competition!

Advance review by James Conway III, OH

When I began reading *Mathematics for Human Flourishing* by Francis Su, I did not know what to expect. Reading a book about math by a math genius is a little intimidating. However, this book is more about the flourishing than the math. It is part memoir and part teaching philosophy with a sprinkling of math. The personal stories and colloquial style pull you in and make you comfortable after the first few pages. At the end of the first chapter you may be slightly confused by the seemingly unrelated strands of narrative. Just keep going. Mr. Su pulls everything together nicely to convey an extremely important message.

From the outset it is clear that Mr. Su belongs to the small subset of the population that loves working in the field of mathematics. The question to the reader, though, is, "What does this mean for the average person?" That brings about the first major point of the book: Actually solving math problems isn't of primary importance. It's the act of reasoning and mental exploration that is truly what is important. No, everyone may not aspire to a degree in mathematics, but Mr. Su demonstrates how each person can benefit from practicing and experiencing mathematical reasoning on a daily basis. As he puts it, "The pursuit of math can... build aspects of character and habits of mind that will allow you to live a more fully human life and experience the best of what life has to offer."

Each chapter ends with a puzzle which can be solved through use of reason and mental exploration. These puzzles take you out of the narrative and very effectively demonstrate how intriguing and beneficial such exercises can be. The key then, according to Mr. Su, to getting everyone interested is presenting mathematics in a very different way than has been done in the past. Problems must promote exploration through real world situations and not simply be abstract exercises in rote memorization. They must seem more like play than work.

The final few chapters deal with who is included in the math community. Essentially, mathematics has been an exclusive field. Race and sex have been used, sometimes blatantly, sometimes subtly, to push people out, and anyone who had trouble learning through rote memorization and repetitive exercises was deemed of insufficient intelligence to handle the subject. This is where the book really hits its stride and where Mr. Su has put the most energy. No one has the ability to look at an individual and know her capacity for mathematics. Not even standard tests can discern the characteristics that are important to great mathematicians such as creativity and perseverance. They test for knowledge which is very different from creativity. In addition, the math community as a whole benefits from diverse perspectives. Thus, the system in place used to decide who enters this world is deeply flawed.

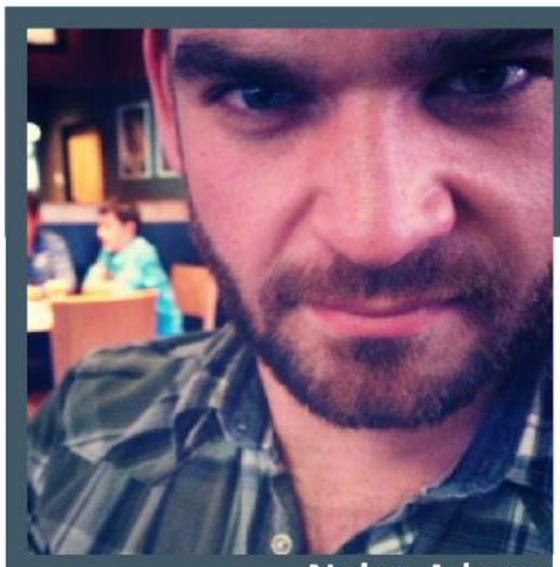
Not only should everyone be given access, it is the duty of society to instill in every individual the highest level of mathematical reasoning of which they are capable. The final point made by this book is that a good mathematics education is essential for the functioning of any society, particularly a democracy. We need it to know truth from deception. And math is the language of information which is critical to making any decision.

Taking down barriers is not enough, argues Mr. Su in the final chapters. Teachers must be proactive in making the classroom feel welcoming and creating an environment where everyone feels free to speak and ask questions. Being in a room with a teacher is not the same as being in an interactive community of people exploring and debating. It may seem petty to some, but trying to learn a difficult subject without feeling comfortable asking questions can kill a math career.

While a damning indictment of the current education system, this book is not apocalyptic. The way math is taught has reduced the ability of society to function and solve complicated problems, and it makes you wonder where the human race would be if education had been more inclusive. Yet, there is still hope. Mr. Su outlines a plausible path for the future. This is a guidebook for anyone who wishes to know how to unlock the true potential of society's greatest asset: *people*.

How to participate: If you can't obtain the featured book on your own, request a copy from the PMP (address on the back page). Once you've read the book, send your reviews via snail mail or email them to us at newsletter@pmathp.org. The trade-off is this: we send you the book, you send us a review. Fair?

Reviews must be no longer than 1,000 words and may be edited for length or clarity. All submissions for *Mathematics for Human Flourishing* must be received by **July 30, 2023**.



Nolan Adams

Claire Finlayson

PARTICIPANT SPOTLIGHT

by Claire Finlayson

Well, someone *has to keep these math geeks in line and jump on every little spelling mistake...*

I'm a writer from BC, Canada, so don't try to bust me for using British spelling, like "cheque" and "colour," okay?

I am a PMP superfan, and my rudimentary math skills have not proven to be an impediment to my participation in this wonderful organization. It's the unique, true-life stories of people that interest me. So I write profiles of participants, volunteer mentors and others involved in the program.

If there's someone you'd like me to spotlight, here's how you can contact me:

www.clairefinlayson.com

claire@pmathp.org

Prison Mathematics Project,
10810 N. Tatum Blvd Ste 102-998
Phoenix, AZ 85028

It's my great pleasure to introduce you to Nolan Adams. Nolan, 33, is a native of Dallas, Texas, and one of our PMP volunteers. He is our nonprofit development coordinator and performs other vital administrative tasks to help take the PMP to the next level.

CF: Hi, Nolan. Can you start by telling me what you're up to these days and how you came to be associated with the PMP?

NA: I first heard of the PMP through a Reddit post. When I'm not restoring vintage furniture, I'm either focusing on my studies or my role at PMP.

CF: I know how busy you are: I had to book a spot in advance on your Google calendar just to get a chance to Zoom with you.

What skills do you hope to contribute to the PMP? What do you see as your role?

NA: I am currently enrolled in the Nonprofit Leadership Studies program at the University of North Texas. This program has provided the education and skills necessary to act as PMP's grant coordinator, along with providing insight into other aspects of non-profit strategy.

CF: When we chatted on zoom, you mentioned a pivotal event in your life that happened when you were 18. You were convicted of theft, right? How did that conviction affect your future opportunities?

NA: That is correct. At eighteen, I was fresh out of high school and in my first apartment. My roommate and his live-in girlfriend weren't able to make rent. They proposed going to her mother's home to pick up some of her signed baseballs to take to a pawn shop. I should've known better when he went through the doggie door. More on that later. That charge has been a relatively constant barrier in my life, standing between me and opportunity. When applying for work, my resume gets tossed to the "bottom of the pile".

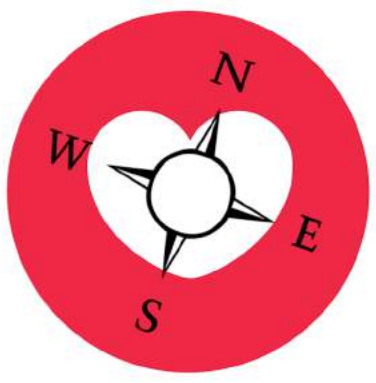
Universities have denied me admission, and finding housing has been a hurdle in the past. In addition, I will never be able to run for office or practice law, which I've dreamed of doing of since childhood. After being shown that others consider you a threat to safety — that you're not worthy of redemption — over and over, you really begin to internalize it. I spent years believing that I was what they showed me.

CF: I'm sure many of our readers can identify with that! What would you like to say to them?

NA: Take my advice: don't believe it. Keep your head up, stay proud, and prove them wrong.

CF: Can you say more about that early encounter with the system, and how it has given you some perspective on the situation faced by those with criminal convictions?

NA: True to the norm, as a white male from an upper-middle-class family I was sentenced to deferred probation. At my hearing I watched on as others from different backgrounds received five, eight, and ten-year sentences for similar offenses. That day opened my eyes to the nature of American justice. The deck is stacked.



CF: You made some comments about how, back when you were a kid, you didn't have the accurate moral compass or adult judgment to determine a healthy friendship vs. unhealthy one. And it was the influence of a friend that got you into trouble in the first place. What do you think about the influence of prison associates on an inmate's future trajectory, and how do you think the PMP could disrupt any bad trends?

NA: I was homeschooled, and in retrospect it wasn't the best environment for me. I valued the autonomy associated with homeschooling, but I longed for the friendships, relationships, and other social experiences of public high school.

I know some of our readers might prefer to leave their high school experience in the past. I don't know whether I would cherish those memories, but the lack of friendships left me unequipped to differentiate between a healthy friendship and an unhealthy one. It didn't take long for me to form an unhealthy friendship with my then-roommate Jeremy.

CF: That's an all-too-common scenario, especially for young people still working out who they are. I bet many people in prison can cite the influence of friends as a powerful driver in the choices they made. But tell me, why are you specifically interested in helping inmates post-release?

NA: There are roughly 45,000 collateral consequences associated with a felony conviction. These 45,000 policies effectively deny 24 million Americans the right to thrive. Post-conviction unemployment results in an estimated $78 to $87 billion in annual GDP, and has disenfranchised 5.17 million potential voters — an outright denial of the most fundamental American rights. An entire segment of our population is pushed into the shadows of society, with little to no reprieve. After gaining a better understanding of the profound impacts of this socio-political disenfranchisement, I feel compelled to take a stand. All of us should be taking a stand.

Dr. Amy Shell-Gellasch

THAT IS SO COOL!

Dr. Amy Shell-Gellasch is a full time lecturer at Eastern Michigan University. She earned her DA in mathematics from the University of Illinois at Chicago in 2000 and followed that with a post doctorate position at the United States Military Academy at West Point, NY. Her area of research is the History of Mathematics and its uses in teaching. She co-founded and currently chairs the History of Mathematics Special Interest Group of the MAA and is an associate editor of Convergence online journal. Currently she is the Chair of the Michigan Section of the MAA. She conducted research on mathematical devices at the Smithsonian National Museum of American History from 2012-2017.

Why 180°?

One of the math facts most people do remember from their school days is that the sum of the angles of a triangle add to 180°. Why is that? What is so special about 180°? This is not a rhetorical question like, "Are you going to take the trash out?" Is there anything special about the number 180? Or about 180°? You think on that while I take the trash out...

Well, there is not anything special about the number 180, though it is an abundant number (its proper divisors add to more than itself.) And degrees (360 in a full circle due to the ancient Babylonians) are a relatively arbitrary measurement system of circles based on a convenient approximation of 365, the number of days in a year, and each degree corresponds roughly to the distance the moon travels through the zodiac in a day. But we could just as easily, and do, measure circles using other units, like radians with 2π radians in a circle and thus π radians in a triangle; if Bonaparte had had his way, we would have 100 decimal degrees in a circle with a triangle angle sum of 50 decimal degrees; or if we measured circles in a baker's dozen of rabbits, a triangle would sum to 6.5 bunnies (don't think about that one too carefully.)

6 Bunnies and a Chinchilla
*(the universal standard measure of half a bunny)**

This is surely cool, but why is it? There is a fun and easy paper and scissors demonstration I like to show my elementary education majors. Grab a piece of paper and scissors and cut out ANY triangle. Go ahead, I'll wait, I need to take the recycling out...

Any triangle

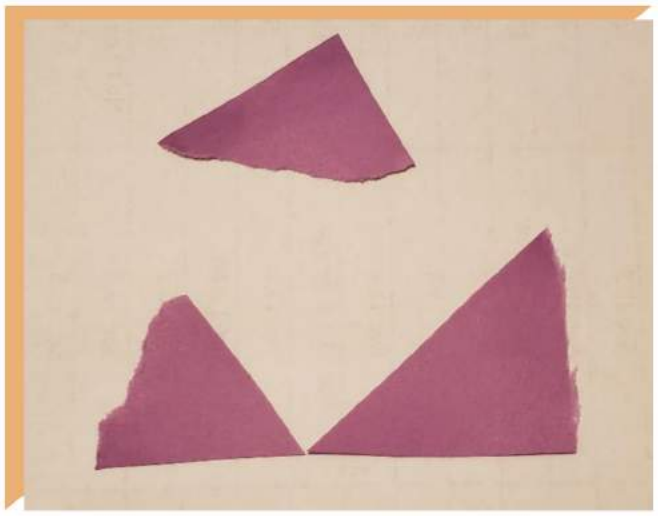
Now carefully rip off the corners

Corners surgically removed

*From left to right: Otis, Murphy, Merry, Pippen, Urszula, Flurry, and Nigel. (All rescues, three are decimal bunnies for better accuracy, being amputees.) There is a Shetland Sheepdog in the house too, but she is used for larger measurements.

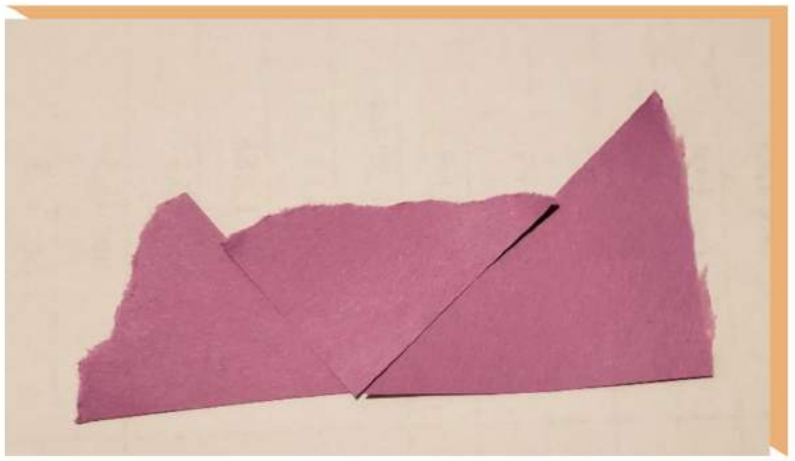
ITERATION 6

Take any two and form a line with the sides, vertices (points) together.



Straight line formed

Now, miraculously, the third vertex will fit in between perfectly.



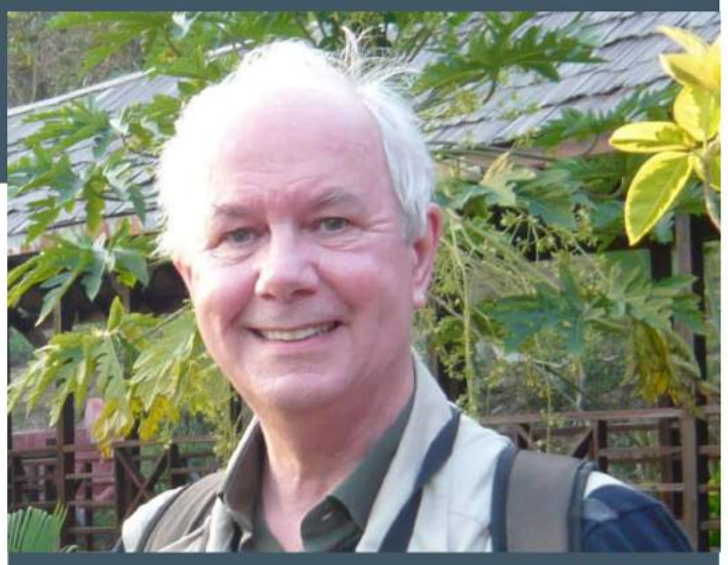
Straight line completed

Angles that add to a straight line form a "straight" angle of 180°. And the straight angle is the angle of the diameter of a circle forming a semicircle. Voila! (or Viola, as I prefer) 180°. Cool!

And if you act now, the first person to donate $180 to PMP gets the original Jammin' triangle pieces as a keepsake. (email your dated receipt to ashellge@emich.edu)

MATHEMATICS FOR THE BILLION

While no job or community is perfectly free from "bad apples," Ian Stewart is here to make the case for math.



Ian Stewart
University of Warwick

Even a tiny improvement in math ability can make everyone's life better. Just a glimmering of interest in the subject can open the door to a new hobby, a more rewarding job... a new life. Math empowers us. It helps us understand the world we live in, distinguish truth from falsehood, and spot poor logic and misleading statistics. We can revel in the subject's inner beauty and what it tells us about Nature. We can leverage those experiences to help us understand the world we live in, distinguish truth from falsehood, and spot poor logic and misleading statistics. We can revel in the subject's inner beauty and what it tells us about Nature. We can leverage those experiences to help us grow as a person. We can contribute more to the welfare of our fellow humans.

We mathematicians love our subject. Of course we do: we're self-selected. Many non-mathematicians also appreciate the importance of math, even though they don't want to become mathematicians themselves. But —

I hate to admit this, but it's hard to deny — most of humanity finds math baffling and pointless. They never use it themselves, they never see anyone else using it. But then... I'm not a surgeon. Give me a scalpel and a patient and I'd probably faint. I've never seen a surgeon at work. Yet my lack of knowledge and personal experience doesn't mean surgery is pointless. So why do so many of us think that about math? Unfair comparison, you say? Agreed, I know lots of people who've benefited from surgery. So I have experiences that prove to me that surgeons are useful members of society. In contrast, most of us think they don't know people who've benefited from math. Actually they do: themselves among them. But they don't know that they know such people, because they've never had any reason to make the connection. We use math by the bucketful, every day, but we never notice because it's hidden from sight.

The word 'mathematics' is used in two very different ways. To most of us, it refers to a limited and sterile series of exercises in calculation, usually with no practical aim in mind. (Well-intentioned questions about a dog and a half digging a hole and a half in a day and a half tend to reinforce this perception.) It's hardly surprising if people get put off by this view of the subject. But there's another meaning: a boundless, hugely creative system of ideas and methods with a history spanning at least three millennia. A subject that illuminates our world, from the tiniest atom to the entire universe. A source of magic and beauty. And a little digging (not by a dog and a half, I hasten to add) reveals that is lies just beneath the surface of the transformative technologies that made the 20th century totally different from any previous era, and are making the 21st century even more different.

I'm going to quote one set of statistics. Some years ago a study for the US government found that the mathematical sciences contributed *37 trillion dollars* to the US economy in the first decade of the 21st century. That's $37,000,000,000,000. If that amount of money were shared equally among America's 320 million people, they'd each get more than $100,000.

Not bad for an allegedly useless subject.

How on earth can math be so valuable? The reason is simple, though it may surprise you. Math is of vital importance to *everyone*. You may not use it in any obvious way yourself, but everything in your life is underpinned by people who do, and the fruits of their labors. Computers, the Internet, cellphones, satellite navigation, airplanes... without massive quantities of math, none of these would exist. Without statistics, modern medicines would never have been invented. Supermarkets wouldn't be able to keep the right goods in stock. The financial markets would grind to a halt. Even the humble potato is — today — a product of a long series of breeding experiments, analyzed mathematically to find out what works best.

You don't need to do math yourself to benefit from these things, but any informed citizen ought to be aware of the vital role that math plays. If you do decide to join the ranks of the people who make such wizardry possible, 3.7 trillion dollars a year pays good salaries. But it's not just about dollars and cents. Math can improve social justice, by making us aware of inequalities and marginalization. It can stop us falling for conspiracy theories. It can enhance our individual sense of worth: anyone who has successfully solved a math problem knows what a boost to confidence that can give. If you're serving a sentence for a crime, an improved grasp of math won't just get you a job after release: it can help you become a more useful member of society.

Lack of public awareness of the utility, beauty, and ubiquity of math isn't a new problem. In 1936 the medical statistician Lancelot Hogben published a classic classic popular science book: *Mathematics for the Million*. His best-seller wasn't even a scholarly text. He wrote it ' in hospital

during a long illness for my own fun', and had it published only when friends persuaded him that would be a good idea. He wanted math to be accessible to everyone. Above all, he wanted to help them use it for themselves. He was both evangelist and educator.

In Hogben's day, a typical citizen did actually need basic arithmetic, if only to check the bill when shopping. Today we have calculators to do the grunt work, and the supermarket checkout totals the bill, adds the purchase tax, subtracts the discount vouchers. We mainly listen to the beeps as the laser scans the barcodes, and as long as the beeps match the goods, we assume all is well. We've outsourced our arithmetic to electronic devices.

That said, we enjoy — and indeed rely on — the fruits of mathematical discoveries to a far greater extent than anyone did between the two world wars. Math is no longer for mere millions: we unknowingly exploit it in our billions. Without vast quantities of math, far more diverse than anything Hogben could have imagined, today's world would cease to function. As we repeatedly discover when the bank's website crashes, the phone's battery runs down, or the engine manager in our car glitches and the stupid thing refuses to start, even though there's nothing mechanically wrong.

Math can help build a fairer society. It can tell us when corrupt politicians are engaging in the time-honoured scam of gerrymandering — drawing boundaries of electoral districts to reduce the effect of opposing voters and increase effect of supporting ones. Math can stop law courts making incorrect deductions from statistical evidence. A well-known example is the prosecutor's fallacy, which occurs when the court is persuaded to assume that two different probabilities are the same. These are: the probability that the defendant is innocent, given the evidence, and the probability of that evidence occurring if the defendant is innocent. Often the second probability is very small, leading the prosecution to argue that the first is equally small.

Two notorious cases of this kind are those of the English lawyer Sally Clark, convicted of murdering her two infant sons, and the Dutch nurse Lucia de Berk, convicted of several murders on the grounds that a blip in deaths coincided with her periods on duty.

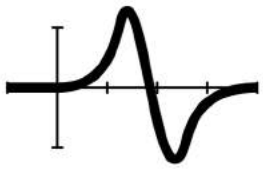
Clark's sons both died from sudden infant death syndrome — 'cot death'. This is the commonest cause of death in the first year of a child's life, occurring for a ballpark figure of one child in 10,000. An 'expert witness' calculated the odds of two cot deaths as one in 73 million. This calculation in effect deduced that the probability of two cot deaths is one in $10{,}000^2 = 100$ million, but used a slightly lower figure than 10,000. This might be true if you pick someone at random, but in this case the relevant population is restricted to families in which there has already been one cot death. The probability of a second, conditional on the first, remains at one in 10,000. Actually, cot deaths tend to run in families, and a second one is about five times as likely, by chance alone. In any case, that's the wrong probability. The correct question is: when a family suffers two cot deaths, what is the probability that their mother murdered them? This is extremely low.

De Berk's conviction also largely rested on an 'expert witness', who testified that the chance of the same nurse being present at all of the deaths was one in 342 million. However, it should have been obvious that when a large number of nurses move between several hospitals, it's quite likely that some nurse's periods on duty will coincide with a specific small set of the deaths, by pure chance. The calculation assumed the nurse had been chosen at random. Actually, she was under suspicion precisely because she was present at those times. It's like arresting someone for winning the lottery, on the grounds that their chance of winning is far smaller than the chance they were cheating. Not so: the chance they've won is a dead cert, because they did. That's why you arrested them.

A later calculation revised the figure to 1 in 25 — but once again they were calculating the wrong thing. Worse, the

calculation presented to the court was founded on the assumption that the deaths were caused by a nurse. If not, the calculated probability of it being this particular nurse is meaningless — the true probability is zero! Later it became clear that there wasn't any convincing evidence that anyone had been murdered. Both women were later — much later — declared innocent and released. De Berk was eventually paid compensation. Clark, who never recovered from her wrongful incarceration, committed suicide four years later. They're not alone in being tragic victims of this kind of innumeracy. So how do we avoid repeating the same errors and destroying thousands more innocent lives?

By becoming more numerate.



How do you go about doing that?

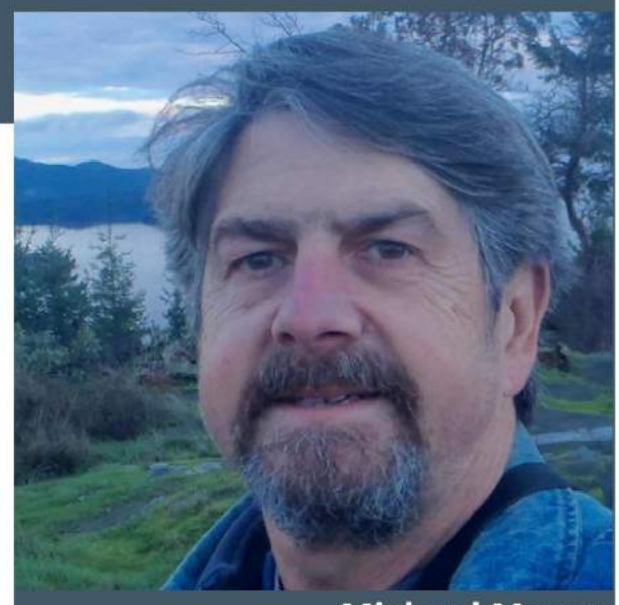
I recommend easing in gently. Learn to enjoy math first, worry about the technicalities later. If your experience at school was a bad one, as it all to often is, even with good teachers, that might be because it was done the other way round. Young children are usually fascinated by shapes and numbers, but somewhere along the line their innate love of these things gets lost. I suspect some of that comes from too much emphasis on learning the methods and getting the right answers. Those things are important — but what math is for, where it came from, and why it's interesting are just as important, and within most people's comfort zone. Once you feel comfortable that math is worth doing, then you start on the sums.

When I was a kid, there was no Internet. To inform yourself, you read books. I haunted the local library, but in those days the number of accessible math books was tiny. Hogben was a statistical outlier. Today, there are hundreds of easy-to-read, comprehensible books; a whole genre of 'popular math'. Reading books like that is still a good way into the subject. And now there's also a huge range of accessible information on the Internet. You just have to search for it, which is easy, courtesy of two math billionaires.

There's something out there to suit any taste. Sources suitable for non-specialists range from the basics to the frontiers of research. They focus on many different aspects of math: its history (many of the famous mathematicians of the past led lives that would grace any TV soap), what it reveals about the natural world (we all know why the rainbow has colours... but why is it that shape?), what it's for (medical scanners, airline schedules, spectacular effects in movies...). You can find out about the hottest unsolved problems (the Riemann Hypothesis, P ≠ NP, the Birch–Swinnerton-Dyer Conjecture), and how some of what used to be the hottest problems have been solved (Fermat's Last Theorem, the Poincaré Conjecture). You can spruce up your basic math if you want to do some real sums. You can win a million dollars by solving one of those hot problems.

What are you waiting for?

LEARN YOUR WAY



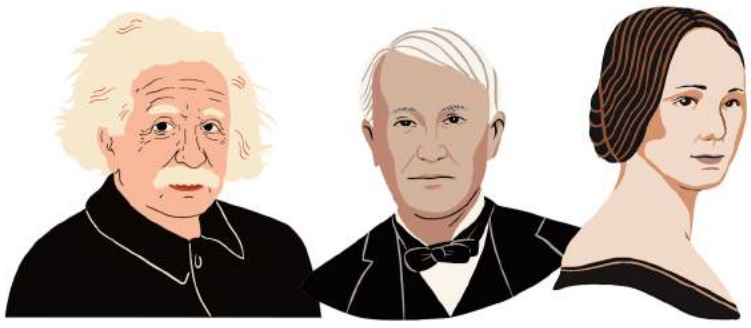
Michael Maser

New to the PMP family in 2023, Michael Maser is an award-winning educator, author and learning coach who teaches Neurobiology and Learning in the Individual Masters program with Antioch University. He's just wrapping up his PhD, focused on learning, at Simon Fraser University, BC, Canada.

His website is www.michaelmaser.net.

Hello and welcome to the first instalment of 'Learn Your Way.'

I hope this column helps you see that you have the same potential to learn math, or anything for that matter, as any 'genius' that might come to mind: Albert Einstein, Thomas Edison, Ada Lovelace ...



I know that might seem like a stretch - Einstein is considered one of the smartest dudes ever, Edison changed the world with his inventions (light bulb, phonograph, etc.) and Lovelace helped pioneer 'computer thinking' 75 years before the first computer was built.

But they all possessed the same 'brainware' as you, and believe it or not, their school teachers thought they were odd and had limited potential. School, in fact, had little to do with helping nurture their 'genius.' That happens too often so it is important to distinguish between schooling and learning. They aren't the same thing.

Recent insights into human learning confirm that each person learns lifelong and uniquely. Yes, we share basic brain structure but no two of us learn in the same way. And most learning and learning potential reflects your life-events and whether or not you're interested in learning something. If you're good at some activity - and everyone is good at something - chances are that is an area of learning 'strength', and that's important to recognize because you can grow that strength and apply it to other things you want to know.

For too long, schooling and mainstream education have advanced bone-headed ideas about learning—for example, that 'intelligence' is best determined by one-size-fits-all curriculum and testing, and who you are as a person isn't important. This limiting belief excludes many people who think 'differently' or tune out schooling. Einstein's teachers said he was 'mentally retarded' in his middle school math class. A few years later he was awarded the Nobel Prize for the 'Theory of Relativity' he figured out while working as a clerk. He later related that in the back of his math class he was daydreaming, but this was actually how he began to 'see' and test his award-winning theory. In his imagination. By thinking differently.

Now I'm not suggesting you might be the next Einstein. But I hope to provide you with some keys to unlock the 'prison of your mind' and 'fly free' to learn the things you really want to learn or didn't know you had the potential to learn.

You start on this path by deputizing yourself as a 'learning detective.'

Learning Detective I: How you are smart. Everyone, including you, does something uniquely well that no one else does. Maybe you were rewarded for such talents in school, and maybe your strengths went unnoticed. That doesn't matter; what does is that you recognize your own strengths - no matter how trivial you think these might be, and how you bring your strengths forward to use now and also incorporate into a new 'I can learn' identity. Let's unpack this a little more and help you discover your genius.

First, identify and write down what you are good at. C'mon, don't hold back! If you draw pretty well, list 'draw;' if you fix stuff pretty well, list 'fix stuff'. If you tell jokes, or imagine fantasies, get angry or solve puzzles quickly, then list this. Don't judge your activities, just inventory them. Now, describe how you do this: Note all the details about this: timing, location and context (and to confirm, this is NOT a test but is for your own information. You don't need to share it with anyone).

Of course, whatever you're good at doing reflects your learning, so let's look at this more closely - let's 'inventory' your 'learn-scape.' Think about what you listed above. How, exactly, did you learn this skill? Watching someone? Trial and error? Both?? Now, identify what influences your learning and doing. If you are more productive at some times than others list why you think this is so: What conditions help or hinder your learning: is it your ability to concentrate or your emotional state? Is it the food you eat? How tired you are feeling? Thinking about past experiences (school, home, etc.)? Is your learning influenced by other things?

Learning is a complex activity. And to optimize your learning potential you must understand your own learning 'ecosystem' to amplify what helps you learn and reduce what hinders your learning. By the way, experts say a little stress helps spark learning but too much stress blocks it.

Thinking about math and learning: Some of you have experienced some success at math. Others might say, "F*** math! I don't think in numbers and algebra or any of that crap!

You can rant about it but I don't believe you. I'm no math whiz myself but I've lived long enough to see that everyone learns and practices math in some ways. Maybe in school you didn't succeed in solving math word problems or algebra equations or geometry, but 'math' is much more than just these things or a course grade.

'Math thinking' is logical thinking: If this, then that. "If it's snowing and I wear shorts I'm going to get cold. So I'm going to wear jeans." It's solving puzzles, playing cards, shooting hoops, fixing a truck, and cooking.

Ever done any of those things? Okay, it's confirmed, you 'do' math. It's time to get beyond past judgments and how someone might have said to you, "you can't do math."

Your homework: I'm going to feed your brain and your soul in this column but you have work to do to get the full benefit of this. Your work is to think about what you're good at - your 'genius' as I mentioned above - and think about how you might apply your 'genius thinking' to other things you want to learn. Think about this! Write stuff down as it occurs to you. And you need to get clear on what you want to learn. Not what you're pissed off at or what doesn't work for you, but your learning goal(s). You can change this later but for right now, get clear on this. I'll have more to share in my next column.

Christopher Havens

A MISCREANT'S MISCELLANY

by Christopher Havens

Christopher Havens in Conversation with Andreas Hinz

Andreas Hinz

CH: Greetings readers! I've often thought that a "prison" by definition is anything that confines one's mind and voice, so they travel no further than some physical boundary. In my own case, that boundary was a roll of razor wire around the penitentiary where I am incarcerated—but I have been freed as a result of my exploration into the world of mathematics. I see this now in the most metaphorical way: everyone has their own version of the razor wire. But if there's one thing I've learned, it's that opening yourself up to experiencing the rich cultures of the things we love, along with the amazing communities that accompany such cultures, can usher you along endless avenues of beauty and meaning, and you meet a hell of a lot of interesting people along the way.

Physically, I sit in a small concrete room, loaded with maths of all shapes and sizes, but in my mind I may be traveling the streets of Rome in search of a secret piece of geometry known only by the Marmorari Romani, or I may be enjoying a summer breeze in Saint-Jean-de-Monts on the west coast of France. And nearby I am watching a discovery form in the mind of a mathematician as he sits under an umbrella in his garden. I refer to these as "adventures" because the mental images are there in my mind, making me feel as if they are taking place in the physical world.

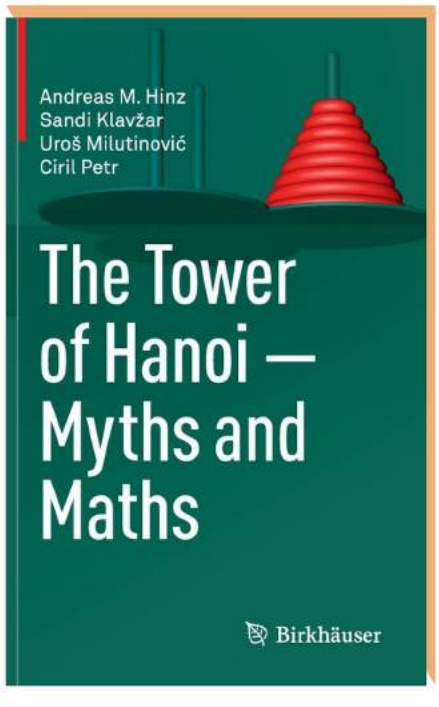
Andreas Hinz is one of the mathematicians I had the pleasure to meet on such an adventure, where I was exploring the history of the Sierpiński Triangle, and so it brings me great pleasure to have him here with us today. Andreas, hello! What an honor to speak with you. How about you start by telling us a little about yourself.

AH: Certainly. I was born in 1954 in the north of Germany and passed my youth in the historic town of Worms on the Rhine River. I studied mathematics, physics, meteorology, astronomy, and the history of science in Munich, where I met my now wife Christine. We have three daughters and four grandchildren. After a one-year visit to Geneva, I defended my PhD and later my habilitation at the University of Munich. I opted for mathematics because of its beauty, universality and independence of ideologies. My teaching and research took me to universities in Cardiff (Wales), Hagen (Germany), and Maribor (Slovenia), as well as to conferences and short visits on all continents except Antarctica.

My main field was mathematical physics, where partial differential equations are employed to model phenomena in quantum mechanics. On the more classical level, I was even able, with the help of a colleague, to improve the mathematical picture of the hanging chain, the *catenary*. When computers became available to me, I encountered the popular game *the Tower of Hanoi* and found a lot of mistakes in its mathematical theory, mainly in the Computer Science literature. So it became my mathematical hobby and I turned to Discrete Mathematics, i.e. combinatorics and graph theory. This is where it came to the episode of Saint-Jean-de-Monts during my family holidays in 1989.

CH: For any who don't already know, Andreas wrote a book called "The Tower of Hanoi - Myths and Maths." It's not only wonderful, but it's the most comprehensive book on the subject out there. Today you are the leading expert in this theory, Andreas, but in 1989 you had just published your first two papers on the topic, am I correct? Can you tell me about the episode of Saint-Jean-de-Monts?

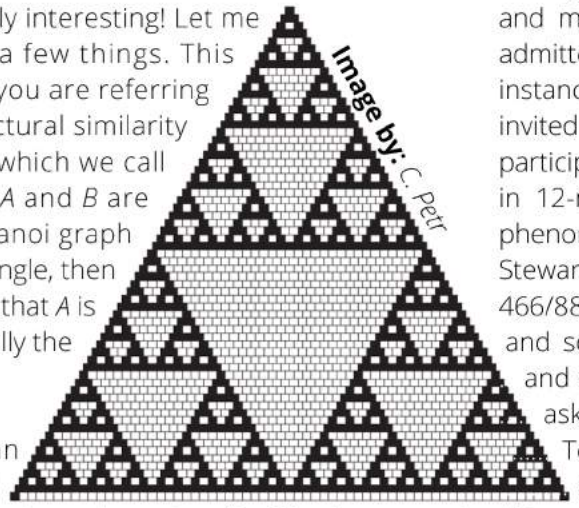
AH: You are right, at that time just two articles of mine on the mathematics of the *Tower of Hanoi* had appeared. In one of them, I used the model of a *graph* for the puzzle. This is not what the reader may know from school to draw a function, but what most "ordinary" people would call a network: Some dots are drawn, every two of which may be linked or not. The distance between two such dots is then the length of a shortest path following these links. For the Tower of Hanoi the so-called *Hanoi graphs* can be used to find the number of moves necessary to get from one state of the puzzle to another one.



One of my main results was a nice formula for the average number of moves—in other words for the average distance in the Hanoi graph.

When we arrived in our holiday apartment in Saint-Jean-de-Monts in France during the summer of 1989, a retired engineer and hobby mathematician contacted me because he had read my paper and casually told me about an article touching on the Tower of Hanoi in the current issue of the French version of *Scientific American*. The author of the article was the important British mathematician and prolific author of popular texts Ian Stewart. He pointed out that the Hanoi graphs are related to a geometric object called the *Sierpiński triangle*, named for its inventor, the Polish mathematician Wacław Sierpiński. It is a standard example of the then very fashionable *self-similar fractals* and is in turn closely related to another triangle, namely Pascal's *arithmetic triangle*, which many of you know from school, containing the *binomial coefficients*. Sitting in a garden chair under a parasol, with my family deposited at the beach, I read Stewart's article and immediately realized two things: There must also be a relation between the Tower of Hanoi and the arithmetical triangle and I could transfer all of my knowledge about Hanoi graphs to the Sierpiński triangle. The former led to an article in the *American Mathematical Monthly*, the latter to the *incredible number* (I. Stewart) 466/885.

CH: Ah! This is incredibly interesting! Let me fill our readers in on a few things. This transfer of knowledge you are referring to is because of a structural similarity between two "things," which we call an *isomorphism*. So, if A and B are two objects, say, the Hanoi graph and the arithmetical triangle, then we write $A \cong B$ to mean that A is isomorphic (or structurally the same) as B.



I have read many of Ian Stewart's papers and books, so this is moving towards a story familiar to me, and probably to many in the math community. So this was where you and Andreas Schief began your collaboration, which established the fact that (the Tower of Hanoi graph) $\cong$ (the Sierpiński graph). Then, knowing the average distance on the n-th Hanoi graph, the isomorphism carries this knowledge to the n-th Sierpiński graph, and then taking the average distance as $n \to \infty$, you obtained the "incredible number" 466/885!

I wonder, would you elaborate on the referee process in publishing this work?

AH: Even with such an outstanding result, it's not easy to get your work published, in particular if you are not an established researcher in the field. Therefore, we asked two experts for their opinion about the average distance on the Sierpiński triangle. The first expert said that it was easy to calculate and the result is 8/15. The second expert found the question very interesting and admitted he couldn't solve it. So we submitted our manuscript to the renowned journal *Probability Theory and Related Fields*, where the second expert was an editor. Since we were able to reconstruct how the first expert could possibly have obtained the wrong value, we included his line of reasoning in our paper: he used a self-similarity argument leading to recursion, but fell into the same trap as the authors of the faulty theories of the Tower of Hanoi which I mentioned before. The paper was readily accepted for publication and well received by the community, and in particular by Ian Stewart, whom I told about it when we met in person in 1990 during the *International Congress of Mathematicians* in Kyoto, Japan.

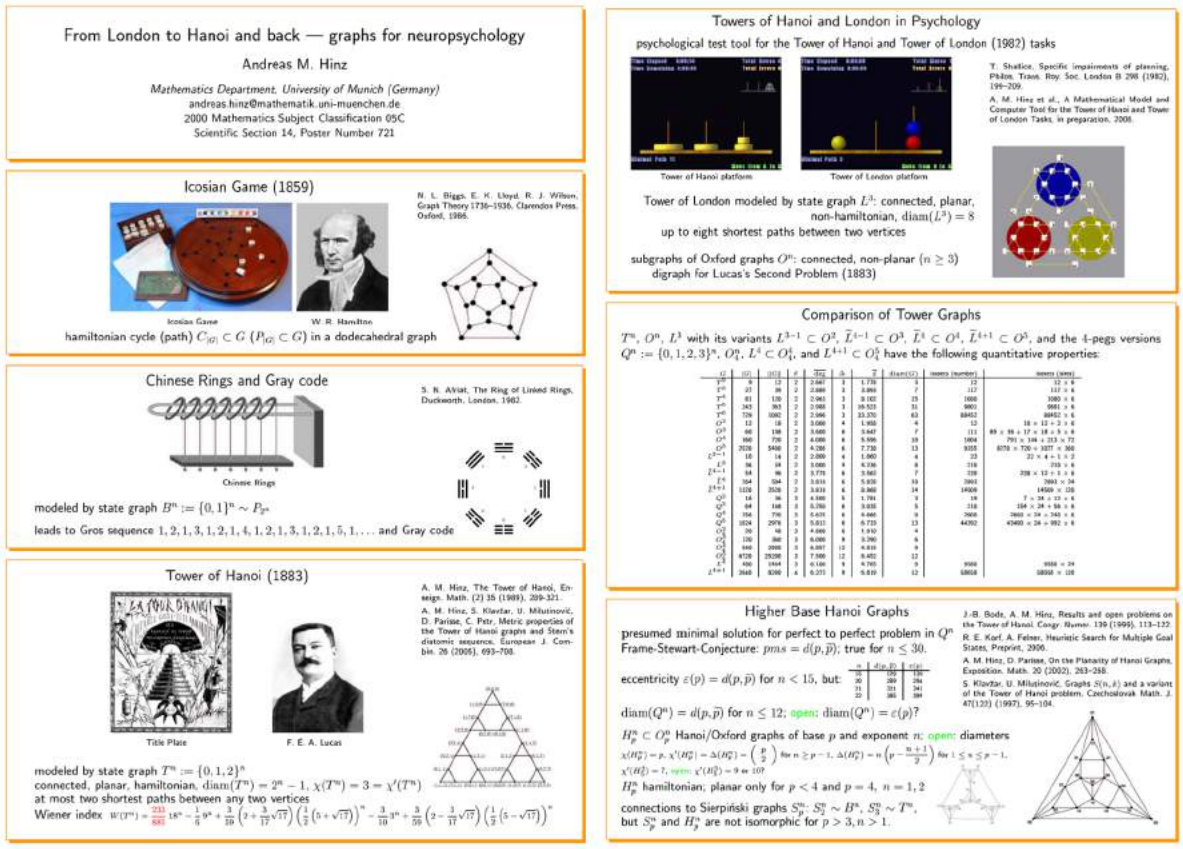
CH: Attending the International Congress of Mathematicians (ICM) is quite an honor! Can you describe to those who have never experienced the ICM what it's like? Also, how did you come about meeting Ian?

AH: The ICM is an exceptional event taking place only once every four years. In our specialized scientific world, mathematics is the only science to hold such a general meeting open to all subdisciplines, which therefore makes it a unique opportunity to obtain at least a slight overview of all developments in the mathematical sciences. The ICMs have been organized since 1897 with obvious interruptions caused by war. Also the 2022 ICM, planned for Saint Petersburg, was only held virtually because of the imperialistic misbehavior of Russia. Having taken part in nine real ICMs since 1983, I am a true veteran, and many of my international contacts originated at ICMs. One is admitted either by invitation or by paying fees. In Kyoto ICM, for instance, there were 15 invited plenary talks of 60 minutes each, 143 invited 45-minute talks, and 651 among the approximately 4000 participants from 76 countries were allowed to present their research in 12-minute slots. My own contributed talk was on a surprising phenomenon in mathematical physics, but I wanted to tell Ian Stewart, whose name I found on the list of participants, about 466/885, so I posted a note on the message board, which he read, and so we met. He was immediately enthusiastic about the result and subsequently made it popular in several publications. When I asked him twenty years later for a foreword to our book on the Tower of Hanoi, he did not hesitate to write a very personal account of his own encounters with the puzzle in the past. Although I never made it to be invited to an ICM, let alone receive one of the prestigious prizes delivered on those occasions, like the *Fields medal*, I nevertheless won a (second) prize (of 100 EUR!) in the poster competition of the 2006 ICM in Madrid, Spain, where I reported on the application of the Tower of Hanoi in psychological tests.

CH: ICM is huge! I imagine there must be rooms upon rooms with writing boards for the exchange of ideas among participants who wish to engage in mathematical chit chat.

I have spoken at a few meetings myself (not ICM) and I have always lacked a true picture of the mood, the atmosphere and the electricity due to my having to appear virtually. Oh, I've felt so incredibly present at some of the events, but I lack the context of physical presence. The closest I've gotten was once in 2017 when I coordinated a Pi Day event where I live now. A whole prison was able to sit in in awe while several mathematicians gave lectures and talks about their research

Andreas Hinz's award-winning poster!



and about mathematical topics. The atmosphere was magical, and my encounters with some of my most influential mathematicians were so animated and engaging that they drew a crowd. For me, it was so special that time seemed to stand still while our mathematical discussions blurred by.

I've read about your and Ian's meeting many times and in many different books and publications, and I have always had a vivid mental picture of what it must have looked like when you and Ian stood together at the Kyoto ICM talking about the Tower of Hanoi. How was it when you met him? Can you describe the mood and the atmosphere in those moments for our readers who have never experienced such an event? I picture you and Ian at a black board writing and speaking while a flock of onlookers gathered. Am I far off?

AH: Well, I don't want to destroy your picture, but the meeting was not quite as romantic. It was the very last day of the ICM, and on my arrival at the site Ian Stewart was waiting for me at the notice board where he had found my message. He is a very kind and serious person and so we did not have to be slowed down by courtesies for too long. I explained my finding very briefly and he understood the point immediately and enthusiastically. I offered to send him an offprint—this was pre-email time!—and we parted ways both quite satisfied: Ian because he had learned an outstanding fact (and an incredible number) and I because I had impressed a well-known mathematician who could perhaps make my result popular. There were no witnesses to our private conversation and as for the availability of blackboards during ICMs, this depends on the location. You can imagine that ordinary university facilities cannot accommodate four to six thousand people easily. In Kyoto we met in the impressive *Kyoto International Conference Hall* (K.I.C.H.), which, of course, was not mainly designed to suit mathematicians. Small intimate rooms are rare in ICMs, so most of the encounters, planned or by chance, are made in the exhibition halls where scientific publishers have their booths, at the sometimes huge dining facilities, or in parks surrounding the venues. Other major attractions are the conference dinners, wine-and-cheese parties offered by publishers or mathematical societies or even embassies and an excursion afternoon, when an armada of buses block the city centers. The sheer density of people creates opportunities for all kinds of discoveries. If the talks by plenary and invited speakers reflect the mainstream of mathematical research, conversations during the coffee breaks offer insight into various activities of ordinary professional mathematicians. I think this is the reason for the term, "congress of mathematicians," rather than "of mathematics."

CH: Well, you described it much differently than I had imagined, but it still seems truly amazing. Four to six *thousand* people! So many encounters and opportunities taking place all around any given person at any given time! Thank you for describing this, Andreas.

My curiosity is piqued around the findings you shared with Ian. Obviously you succeeded in impressing him with your results because it became very popular, and in fact, Ian described this meeting in Kyoto quite often in his books and publications. I'm trying to think of the first occurrence of the story of your encounter at Kyoto and the incredible number 466/885. I first read about it in Ian Stewart's article, "Four Encounters with Sierpiński's Gasket," which is a must-read for anyone interested in Ian's side of the story. But if I recall, didn't it also appear in the book "*Another Fine Math You've Gotten Me Into*"? What I wouldn't give to be a fly on the wall at your place to see the expression on your face when you first read about your work in popular mathematics publications. There's nothing like the feeling of reaching a large number of people with something so close to your heart, yes?

AH: The reason why my mathematical gem was met with enthusiasm by Ian Stewart is that it broke open doors. He had already studied both the graph structure of (approximations to) the Sierpiński triangle and the Tower of Hanoi, but didn't know my formula for the average distance of two distributions of the discs. On the other side, I was not aware at the time of the fractal structure of the Sierpiński triangle, but after I read Ian's article under French skies, I had all the ingredients to put these things together. Another reason why Ian Stewart took the story up on many occasions, as in the book you mentioned and in "How to Cut a Cake", is that the result is surprising with a "good" argument for the number being 8/15 instead of 466/885 if one approaches the question carelessly. So it also has an educational touch. Of course, I was proud to learn that I've got Ian Stewart into another fine math! To get one's mathematical results published in a decent journal is one thing —and it is not easy. When I sent my first paper on the topic to a Scottish journal, they found out after eight months that it was too long! So someone else, Tat-Hung Chan, could come forward with the Tower of Hanoi formula virtually at the same time in a computer science journal. The moment which you describe as a fly on my wall came for me in the public library of the *Centre Pompidou* in the very center of Paris where I held the volume of *L'Enseignement Mathématique* in my hand with my article in it, visible to the whole world. The twist with the famous fractal and Ian Stewart's advertisements with my name in the author index of his books is another thing and opened my way to a larger audience of even non-professional mathematicians. Also inside mathematics this had an impact. Graph theory is nowadays taken much more serious than three decades ago, say. And many papers about what we now call *Sierpiński-type graphs* appeared meanwhile.

CH: It does have a nice educational touch to it! In fact, I think that introducing discrete mathematics and graph theory in the context of the Tower of Hanoi would make for an incredibly memorable entry into those subjects, if done right. Recently the PMP has been assembling a team of educators to build unaccredited courses for our participants, and this is actually something we've spoken about amongst ourselves as an alternative to what you might commonly find in schools. What can I say... the Tower of Hanoi and its relationship to the Sierpiński triangle - and also Pascal's arithmetic triangle - is just filled with too many beautiful twists and turns, and as you pointed out, there are nice educational touches to it!

In the mean time, what would you say to our readers who wish to delve into the beauty of this topic?

AH: I would certainly encourage them to do so. The topic offers challenges on all levels—from the elementary, like the little exercise you can find in the problems section, to notorious open questions as, e.g., the so-called *Frame-Stewart conjecture*. (Not the same Stewart!) The latter is waiting for a decision for more than ninety years. Our book mentioned before was designed to be elementary, at least at the beginning, which means that it doesn't require too many prerequisites. Nevertheless it contains most of the research on Tower of Hanoi-related topics up to 2017. This field is a microcosm of mathematics and the book is a microcosm of discrete mathematics.

CH: Thank you so much, Andreas, for the insight into your work and for giving us a refreshing taste of mathematical culture. I've always been drawn to the aesthetic appeal of your work.

And for all of you readers, we've given you a version of a treasured story that has been out in the math world since the 90's. Many of you may have already read it in some of Ian Stewart's writings, but if not, go find a copy of the article "*Four Encounters with Sierpiński's Gasket*" from the web, a school library or a friend. Having both perspectives of the same story truly makes you feel a part of it all!

Thanks again Andreas for giving us a taste of your mathematical world. It's always a pleasure.

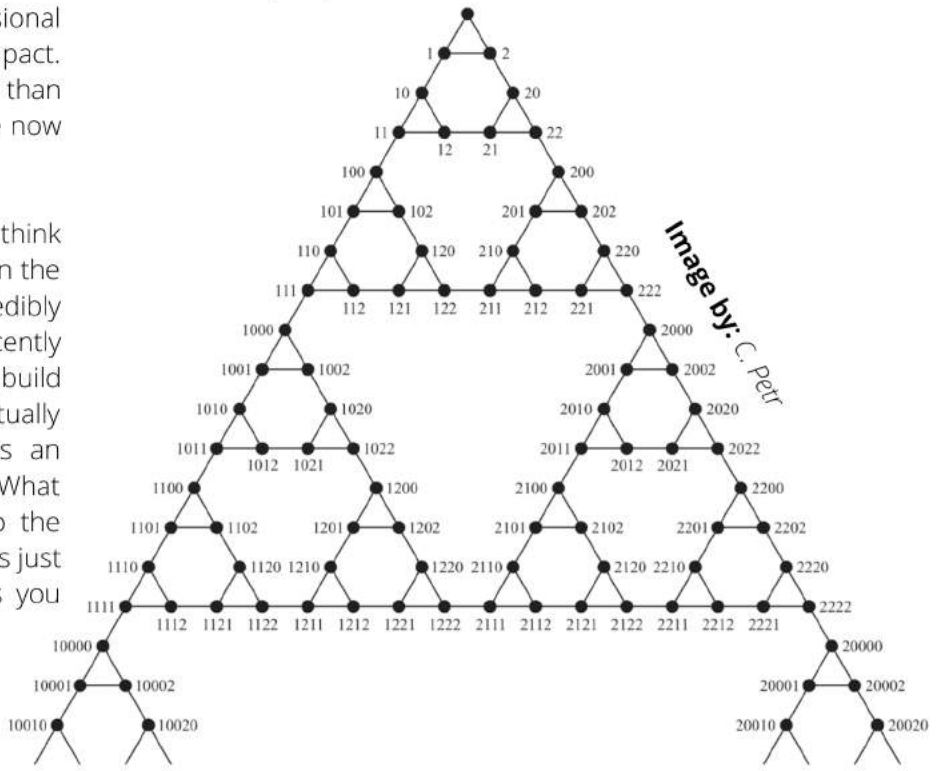


Image by: C. Petr

CLOSING COMMENTS

Ladies and gentlemen,

I trust that all of you had a wonderful Pi Day this year! During the pandemic PMP began hosting live streaming events accessible to all on our website (www.prisonmathproject.org - where some inspiring archival footage is still posted), but this year we decided to extend our outreach efforts by bringing Pi Day INSIDE of the prisons. While it did feel a little odd not having our website blaring the magnificent rants of the world's best mathematical minds for ten hours straight, we are proud to say that Pi Day in Prison was a huge success.

Christopher Havens

The PMP team would like to recognize Bellamy Creek Correctional Facility in Michigan, its administrative and educational staff and prisoner population for a magical PMP Pi Day 2023. Our event was hosted by none other than the Math Guru, Vanessa Vakharia and the cast: David Kochberg and Sabina Wexler, of her popular web show, Math Therapy. We also sent along several mathematicians including Lloyd Douglas, Ben Jeffers, Gary Gordon and Sunil Singh to share what we love about mathematics.

All inmate participants at the event have requested further engagement with PMP.

If you are a resident or staff member at a correctional institution and you would like PMP to host a STEM event at your prison, have one of the administrative staff contact Norton Ewart at norton@pmathp.org. We would be happy to share our love for mathematics with your general population at no cost to the institution.

Some of the PMP Team at Bellamy Creek Correctional Facility in MI

Until next time, we wish all of you a most wonderful day

Christopher Havens and the PMP Team

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