PMP

PRISON MATHEMATICS PROJECT

THE PRISON MATHEMATICS PROJECT NEWSLETTER

FALL 2022 – ITERATION 5

IN THIS ITERATION

Welcome to our latest iteration of the Prison Mathematics Project newsletter! We celebrate inclusivity and diversity in the math community by reaching all lovers of mathematics, especially incarcerated people.

Our writers, editors and contributors from around the globe have designed this newsletter to share with you our deep appreciation for the wonderful world that can be discovered through the study and exploration of mathematics. We connect prisoners who are dedicated to change with mathematical mentors, and through this process, we introduce them to a community that provides an essential framework for rebuilding their lives during their incarceration. The aim of the Prison Mathematics Project is to provide opportunities for our participants to experience a new lifestyle, a new culture that leads to human flourishing through the transformative power of a passion for mathematics.

ASK THE MATH GURU

Do numbers freak you out? Does math send you into meltdown-mode? Are you wondering why we need to learn this stuff ANYways?? Don't worry, The Math Guru is here to help you work through your math trauma, one problem at a time. Ask for advice, guidance, or just a good ol' pep talk! You got this!

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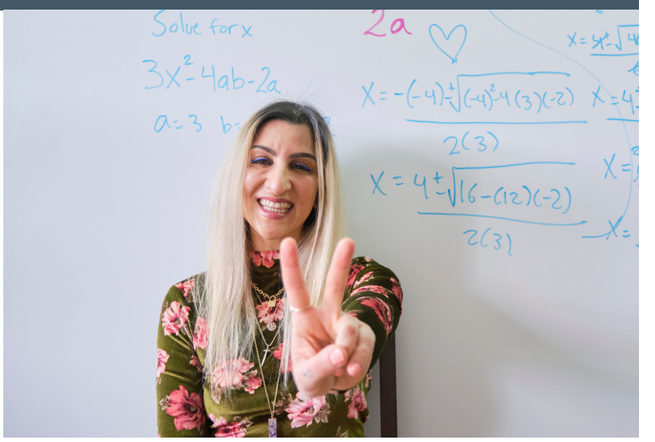

VaneSSa Vakharia,
The Math Guru

Dear Vanessa,

I know you have made it your life's work to vanquish math anxiety, so I'm going to out myself here. Even though I'm the editor of this math-focused newsletter, I worry that, at age 65, it's too late for me to improve my math skills. Maybe I'm just too old and/or dumb to learn anything new. I look in awe at the beautiful symbols our contributors use and I hear the exotic language of mathematics they speak, but it's like gazing at a Michelangelo's David or listening to someone playing Chopin's Nocturne in C Sharp Minor on the piano: I can admire it, I just can't do it. I even have your *Math Hacks* book for ages 7-12. It has been sitting on my night table for a year, but I worry that if I open it, my worst fears will be confirmed. I would like to up my game. I don't want to believe I'm hopeless, but I am literally afraid to start. Anything you can do to help?

Claire F.

Claire!

Thank you so much for your thoughtful and vulnerable question! It takes a lot to dig into the feeling that you just can't do something no matter how hard you try — it's a feeling that a lot of us have, but often don't know how to talk about! The truth is: there's likely nothing on this planet that you 100% CANNOT DO. Sometimes I wonder, for example, if I could become a professional surfer if I started right this second. The truth is, maybe I wouldn't become a PROFESSIONAL surfer...but I could likely learn how to surf even though it seems impossible and I'm very uncoordinated and I don't know the first thing about surfing. I think the same is likely true for you and math! If you feel like conquering ALL of math is too overwhelming and impossible, it's because well, it is! How about instead, you pick something small to tackle? Perhaps you could learn ONE concept like how to calculate a sale price, or how to multiply fractions, or how to convert from degrees Celsius to degrees Fahrenheit. Pick ONE concept, and find a way to learn it. Perhaps that might mean asking a friend to teach it to you. Or maybe that means watching a free math video online or searching about that topic on Google. Once you've mastered that ONE concept, sit back, and be PROUD OF YOURSELF! This is proof that you're capable of more math than you think. This might take 10 minutes, 10 hours, or 10 days. Stick with it until you get it, and once you've recovered from celebrating how smart you are, pick another math topic to tackle. As they say: Rome wasn't built in a day, and most math problems weren't solved in a day either :) Hope this helps, keep us all posted on your progress — you've got this!

☮💗π

VaneSSa Vakharia

Got math anxiety? Think you're a hopeless case? VaneSSa to the rescue! The Lady Gaga of mathematics will put her Master's in Mathematics Education to good use by delivering the ultimate personalized pep-talk!

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INSIDER INSIGHTS

In this column, we publish feedback from PMP participants, mentors and volunteers. Do you have something to say that will educate, encourage, entice or entertain your fellow math students? Drop us a line at the address on the back page, or if you have access to email, send a message to PMP@pmathp.org. We welcome your input.

A letter from Paul Morton, Woodbourne Correctional Facility, Woodbourne, NY

Christopher,

You are the reason I initially wrote to PMP. Somebody here who knows I am into math gave me the Popular Mechanics magazine that had the article about you.

Most people in prison take up hobbies as a way to pass the time, and when they are released they forget about them. For me, mathematics has become a way of life, it's been a part of me for over five years now, and I can't imagine life without it.

Back in early 2017 I was reading an "intro to physics" book, but I didn't understand any of the math, so I found a beat-up beginner algebra book, started on page 1 and never stopped. Something happened to me—I found happiness, true joy, something I realized I had never felt before. I haven't put the math books down since. They have become my secret garden, and I love playing in it.

I came to a crossroads in my studies about a year ago. I had just finished vector calculus and had already gotten my next set of books and solutions manuals for linear algebra and differential equations. But something was bothering me. I just felt that as I turned the last page in my calculus book I hadn't got everything I was supposed to get, there were just too many exercises in the book that I struggled with. For me, somebody with OCD, I needed to go back through it from page 1 right through to the end. That proved to be the right decision. The second time around things made more sense. Things I struggled intuitively with initially I was able to grasp very easily. So 6 months ago I realized that if someone like me who spends 3 hours a day, every day, with math had problems, somebody who is studying for a college course is likely having the same problems. So I started writing a book, and I should be done with it in 6 to 9 months. I have named it *MATH-SENSE: Insights and Intuitions of Differentiation and Integration through Applications*.

I focus on single-variable calculus in this book, and I create problems around concepts and ideas that may be a bit difficult for most people to understand. This book is something I am very proud of. Do you have any idea how hard it is to learn calculus without being able to ask anybody a question? Nobody here knows calculus.

I should be moving on to differential equations, linear algebra and abstract algebra, but I have been "loitering" in calculus. It's been close to a year since I finished vector calculus, but my book project has

me still in calculus. No complaints, though! I truly love calculus – it's sort of magical to me. This whole branch of mathematics is based on the limits of two numbers: zero—which has no value—and infinity—which doesn't exist. Someone was once quoted as saying that "calculus is the language God speaks."

Rob, my mentor, is a student at the University of North Carolina. He started with me when he was 18. Rob's what I call a "high speed student," doing like 21 credits per semester, tutoring other students—and me. I love Rob, he is such a great kid, and so very smart for his age. He's been a big help because he has that "math gift." He just knows. He always has the answers to my questions and explains them in a way that is easy to understand.

My release date is 8/05/23. That is tentative, because I need to find a place to live. I have been writing a lot of letters. I know you can relate. I am not sure at this point what I am going to do, or where I am going to go, but yes, without any doubt, continuing my journey with mathematics will definitely be a top priority in my life. Besides I already have my next set of books to study! :-)

Thanks Chris, thanks for being there.

Paul Morton

THE PRISONER'S DILEMMA

Glen Whitney, AKA The Problem Warden

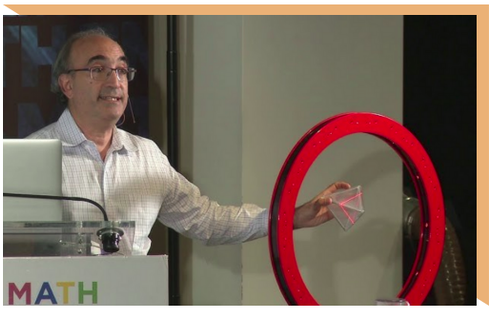
From the Problem Warden

I am delighted to become part of the Prison Mathematics Project through The Prisoner's Dilemma*, as its (honorary) "Problem Warden." My love of mathematics – and especially sharing the joy it can bring – have been a part of almost everything I've done as a professor, financial analyst, parent, founder of the National Museum of Mathematics, author of the Studio Infinity blog, and the most recent ex-editor of* The Playground *(the problem column of* Math Horizons*, a magazine chronicling the world of math for the Mathematical Association of America). I look forward to seeing all of the new methods and creative questions that you come up with as we face the many dilemmas to come, together. And don't worry, as a one-time specialist in mathematical logic, this Warden will keep a sharp eye out for any infractions in your reasoning!* **– Glen Whitney**

By this iteration, hopefully you're familiar with what to expect, but this time we have three new Dilemmas for you to chew on. We have contributions from two new problem posers, one a PMP participant and the other a university professor. And the second two problems feature a mini-theme on ellipses: the elegant ovals you can get by stretching a circle uniformly in one direction.

D7: Three and a Half Sides

Prof. Jim Propp of the University of Massachusetts - Lowell has been corresponding with the Prisoner's Dilemma about planar cross sections of three dimensional solids. For example, suppose you have a regular tetrahedron (a triangular pyramid on an equilateral triangle base with the proper height so that all of the faces are equilateral triangles). If you pass it through a cutting plane so that one face is parallel to the plane, all of the cross sections are triangles, which does not seem surprising. You can see such a cross section, and Prof. Propp himself, in the image to the right. However, if you rotate the tetrahedron so that both of a pair of opposite edges are parallel to the cutting plane and then pass it through the plane, all of the cross sections are rectangles, as shown in **Figure 1**.



Professor Jim Propp giving a demonstration about cross sections of a tetrahedron

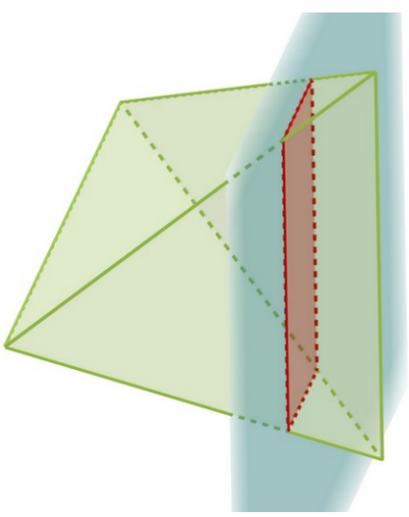


Figure 1: *How to get rectangular cross sections of a tetrahedron*

Can you find an orientation of the tetrahedron that 's exactly in-between? In other words, an orientation with respect to a cutting plane so that when you move the tetrahedron at a constant rate perpendicular to that plane, the *average* number of sides of the cross-sections formed is three and a half?

Prof Propp would contrast these possibilities for a tetrahedron with the situation for another Platonic solid: he has proven that no matter how you orient a cube, when you pass it through a cutting plane in this way the average number of sides of a cross section is always exactly four (even though any number of sides from three to six can occur for a single specific cross section). An octahedron (the solid with eight equilateral triangular faces made by gluing two square pyramids by their square faces) is more like a tetrahedron in this way: if you slice it perpendicular to a line joining two opposite vertices, all of the cross sections are squares, but if you slice it parallel to one of the faces, all of the cross sections (except the lone initial and final ones) are hexagons. What about the other two Platonic solids, the dodecahedron with twelve pentagonal faces and the icosahedron made up of twenty equilateral triangles?

D8: Mullipse?

Recall that given two points in the plane, an ellipse can be defined as the collection of points whose sum of distances to the given points is constant. But here in the Prisoner's Dilemma, we know there are more operations to investigate than just addition (remember Dilemma 4, which appears again below?).

So suppose you are given points F and G a distance two units apart in the plane. We write just XY for the distance between points X and Y in the plane (so for example, FG = 2). Describe the collection of all points M in the plane such that FM · MG = 1. As a start, **Figure 2** shows five of the points in this collection (four of which also happen to lie on the orange ellipse defined by FA + AG = 2.5). You might also want to consider how the shape of this collection changes when you use a constant other than one; in other words, what does the collection of points P such that FP · PG = *c* look like for other values of *c*?

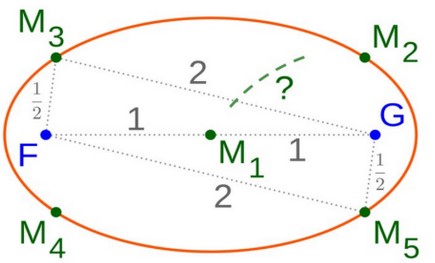


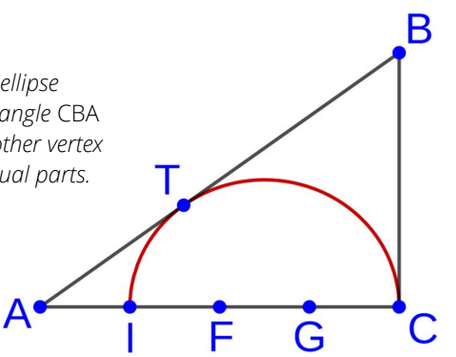
Figure 2: *Five points (in green) on a curve of "constant distance product." Where does the curve go between these points?*

D9: Semi Inellipse

Contributed by Paul Morton, PMP

PMP participant Paul Morton has also investigated the properties of ellipses. As shown in **Figure 3**, semiellipse CTI is inscribed in right triangle CBA so that it is tangent to hypotenuse AB at T, and so that points I and the two foci F and G of the ellipse cut leg AC of the triangle into four equal segments. What is the ratio of the length of hypotenuse AB to the length of leg AC , or in symbols, AB / AC ?

Figure 3: *Semiellipse inscribed in right triangle* CBA *so that its foci and other vertex cut a leg in four equal parts.*



SECOND HEARING

Here's the portion of the column in which we revisit prior dilemmas, to share solutions that readers have come up with, or report on their status.

D4: Four Operations Revisited

(First appeared in Iteration 3, Winter 2022)

This problem challenged you to describe the long-term behavior of four different sequences, each of which used a different one of the four basic operations on the previous two terms to determine what to add to the previous term. Here "long-term behavior" could mean an expression for the *n*th term that just involves *n*, not previous terms. If there doesn't seem to be a closed form, it could be a formula that works when *n* is bigger than some number, but might be off for the first few values. Or sometimes you can find a formula that is never exactly right, but gets closer and closer as *n* gets large. All else failing, it could be one formula that's always less

than the sequence and another that is always greater – and hopefully you can get those two "bounding" formulas as close to each other as possible.

The four sequences, with their recurrence relations rewritten last time symbolically, were:

- $c_{n+2} = c_{n+1} + c_n - 1 + c_{n+1} = 2c_{n+1} + c_n - 1$
- $d_{n+2} = d_{n+1} - d_n + 1 + d_{n+1} = 2d_{n+1} - d_n + 1$
- $e_{n+2} = e_{n+1}e_n/2 + e_{n+1} = e_{n+1}(e_n + 2)/2$
- $f_{n+2} = \lceil 2f_{n+1}/f_n \rceil + f_{n+1}$

Also last time, we saw the solution for sequence (*c*), namely that its entries are sums of entries in a well-known sequence called the Pell sequence, and it has values approximately

$$c_n \approx \frac{(1+\sqrt{2})^n - 1}{4}$$

Solution to D4:

This time PMP participant Jesse Waite of Fort Leavenworth is back at his sequence sleuthing. He notes that examining the first few values of the *d* sequence yields 1, 2, 4, 7, 11, 16, 22, 29, 37, 46, ... He then looked at the differences between successive numbers on this list, i.e., 2 - 1, 4 - 2, 7 - 4, and so on. That produced 1, 2, 3, 4, 5, 6, 7, etc. -- a very familiar pattern! So the *d* sequence must be related to the *triangular numbers*, which are the sums of the first *n* natural numbers. The first six are 0, 1, 3, 6, 10, 15, so we can guess that each of the *d* entries is one more than the corresponding triangular number. Since the formula for the *n*th triangular number (starting from the first being 0) is $n(n-1)/2$, we conclude that the formula we are looking for in this problem is:

$$d_n = n(n - 1)/2 + 1$$

And indeed we can check this by verifying that it satisfies the specified recurrence relation:

$$\begin{aligned} 2d_{n+1} - d_n + 1 &= 2 + n(n + 1) - 1 - n(n - 1)/2 + 1 \\ &= (n + 3n)/2 + 2 \\ &= (n + 3n + 2)/2 + 1 \\ &= (n + 1)(n + 2)/2 + 1 \\ &= d_{n+2} \end{aligned}$$

Jesse also cracked the fourth sequence. Again, examining the first several terms was the key: they are 1, 2, 6, 12, 16, 19, 22, 25. etc. In this case, those successive differences seem to be settling down to always be equal to three. And looking at the recurrence verifies this: once two successive entries f_n and f_{n+1} are at least seven and differ by three, then f_{n+1} / f_n is between 1 and 1.5. Therefore $2f_{n+1} / f_n$ is between 2 and 3, which means $f_{n+2} = 3 + f_{n+1}$, and so the next pair also differ by three (and are still at least seven) and the pattern continues. Hence, as Jesse concluded, $f_n = 3n + 1$ for $n > 4$.

That leaves just the third sequence e_n which has eluded the investigation so far. We will accept submissions for part (*e*) until the deadline for this issue (see below). ■

D5: Triple Fever

Contributed by Ian Stewart, University of Warwick

Caught up in the excitement of the Derby and the Preakness, a buddy of yours heads to the track and discovers that today is the running of the Medium-Rare Stakes. Only three horses

have been entered: Tee Bone, New York Strip, and Rib Eye. You buddy then notices the posted odds: Tee Bone is paying 4-1, New York Strip is at 3-1, and Rib Eye at 2-1.

"Hmm," thinks your friend, "that sounds too good to be true!" Knowing your math prowess, your pal manages to get you on the phone and tells you the situation, wrapping up with "And I only have $94 on me and no time to get more before the windows close. I want to make sure I get this right. Tell me how I should place my wagers!"

Assuming your friend can only buy tickets to win, but can back any or all of the entrants with any amount as long as the bets total $94 or less, how should the money be placed to maximize the winnings *in the worst-case scenario*? In other words, what bets will *guarantee* the most winnings, *regardless* of which horse wins the race? (And how much money will your buddy end up making?)

Solution to D5:

We received solutions from PMP participants William Jones and Sonny Kim. Imagine your bets on Tee Bone, New York Strip, and Rib Eye are t, n, and r dollars, respectively, where of course $t + n + r = 94$. Then, if Tee Bone, New York Strip, or Rib Eye wins, you will end up with $5t$, $4n$, or $3r$ dollars, respectively. That means your total winnings in each of the three cases will be $5t - 94$, $4n - 94$, or $3r - 94$. Therefore your worst-case outcome will be the smallest of those three numbers. But suppose one of them is actually less than the others. That would mean you could *improve* your worst case by betting a little more on that horse and a little less on the others. In other words, we can see that in your best strategy, it must be the case that $5t - 94 = 4n - 94 = 3r - 94$. Now just add 94 to these equations to see that we want $5t = 4n = 3r$. Multiplying by 3 first and then by 4, we see that $15t = 12n$ and $20t = 12r$. These relations suggest that we should multiply our first equation by 12 to ease substitution:

$$12 \cdot 94 = 12t + 12n + 12r = 12t + 15t + 20t = 47t$$

Dividing by 47, we see that $t = 24$, from which we then determine $n = 30$ and $r = 40$. And indeed, these bets total $94, and we see that no matter which horse wins, we end up with $120 ($120 = 5 \cdot 24 = 4 \cdot 30 = 3 \cdot 40$). That means your buddy has guaranteed winnings of $120 – 94 = $26 — not too bad for a day at the races!

In addition, Mr. Brooks and Mr. Sokiera of Loretto solved a different, but related problem. They sought to maximize their possible winnings while guaranteeing that they would not in any case lose any money. In that case, they calculated that to the nearest dollar they should bet $38 on Tee Bone, $24 on NY Strip, and $31 on Rib Eye. That's $93 bet (and $1 kept), and at worst Rib Eye wins and you break even, and at best Tee Bone wins and you end up with $191 for a $97 profit. ■

D6: Rectangle Pinwheel

Contributed by Arsalan Wares, Valdosta State University

The outermost polygon shown in figure 4 as the region shaded in pink consists of 16 congruent rectangles placed so that their edges align (and vertices of four of the rectangles coincide at the center of the polygon). We call this polygon a

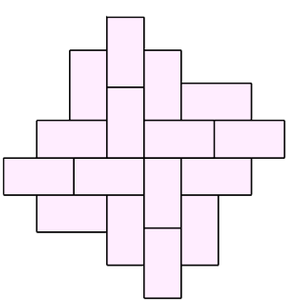


Figure 4: *A pinwheel constructed from 16 rectangles*

28-gon because it has 28 *edges*: the line segments that make up its boundary. The perimeter of each rectangle is 26 cm and the perimeter of the 28-gon is 136 cm. What is the area of the 28-gon?

Solution to D6:

We received solutions to this Dilemma from William Jones and Jesse Waite and partial solutions from Christopher Havens (PMP Founder) and Sonny Kim. As Jesse pointed out, the rectangles *look* like dominoes (length twice the width), but since we want the exact answer, we'd better not rely on that. Instead, let the length of each tile be L and the width be W. We know the perimeter of each rectangle consists of two lengths and two widths, so we conclude that $L + W = 13$. Next we need to use the fact that the perimeter of the entire pinwheel is 136.

Again following Jesse's notes, since the pinwheel has fourfold rotational symmetry, we can look at just one quadrant of it, which must contribute 136/4 = 34 to the perimeter. But the border of that quadrant consists of three widths, one length, and three segments formed by subtracting a width from a length. In symbols, this means

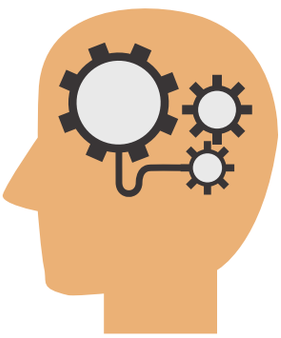
$$34 = 3W + L + 3(L - W) = 4L,$$

so we conclude $L = 17/2$. Now from the rectangle perimeter condition we get $W = 9/2$ (slightly more than half of L, justifying our caution!).

Finally, the total area of the pinwheel is (in units of square centimeters, of course):

$$16\,LW = 16\,\frac{17}{2}\,\frac{9}{2} = 612$$

■



Submission Guidelines

Solutions to problems published in *The Prisoner's Dilemma*, and proposals for new dilemmas, are welcome. For solutions, please clearly indicate the Dilemma number being solved. If a problem has multiple parts, you may submit solutions to any individual part or parts. Solutions to the Dilemmas in this newsletter must be received by the deadline of **2023 March 1**. Dilemma proposals will be considered on an ongoing basis. All submissions should be addressed to Glen Whitney/Prisoner's Dilemma either by email at dilemma@pmathp.org (in which case PDF format is preferred, if possible, although any reasonable format will be accepted), or by mail at:

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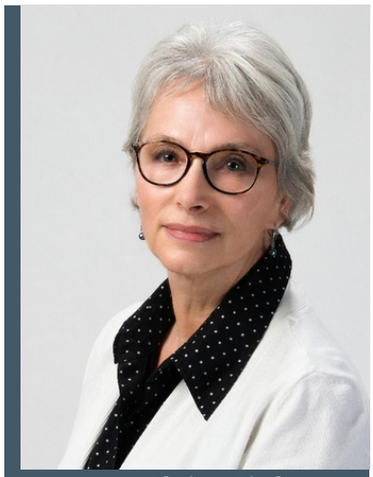
PARTICIPANT SPOTLIGHT

by Claire Finlayson

Jesse Waite



Claire Finlayson

> *Well,* someone *has to keep these math geeks in line and jump on every little spelling mistake...*
>
> *I'm a writer from BC, Canada, so don't try to bust me for using British spelling, like "cheque" and "colour," okay?*
>
> *I am a PMP superfan, and my rudimentary math skills have not proven to be an impediment to my participation in this wonderful organization. It's the unique, true-life stories of people that interest me. So I write profiles of participants, volunteer mentors and others involved in the program.*

If there's someone you'd like me to spotlight, here's how you can contact me:

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For our fifth iteration of the Prison Mathematics Project newsletter, I am branching out from the PMP's home base in Washington State. I have the pleasure of interviewing Jesse Waite, an inmate at Fort Leavenworth, Kansas.

CF: So, Jesse, please tell us a little about yourself.

JW: I am from Ridgecrest, CA, and joined the army in 1995 right out of high school to become a missile maintenance technician on the Bradley Fighting Vehicle at Fort Hood, TX. After six years, one deployment to Bosnia and one overseas tour in Korea, I left the army as a sergeant in 2001. That same year, I enlisted in the air force as a radio frequency communications technician. Over the next twelve years the air force sent me to Tinker AFB, OK; Camp Red Cloud, Korea; Morón AB, Spain and Edwards AFB, CA.—and for good measure, a deployment to Iraq/Afghanistan. My jobs included deployable satellite communications, weather

forecasting, equipment repair, optical telescope maintenance, and logistics test and evaluation for the Global Hawk Unmanned Aerial Vehicle –

CF: That is quite the resume. I first saw your name in Iteration 2 of our newsletter. Your question about inequalities used in a proof or used to solve a problem was answered by PMP founder Christopher Havens in the *Problem Children* column.

JW: Yes, I received that issue and was shocked with excitement when I saw my question published and answered by Christopher! I never thought about using the AM-GM inequality as a means to fine-tune a square root approximation. Christopher's explanation made total sense to me, and I hope it helps others.

CF: I hope so too. I'm glad Christopher whipped your problem child into shape. As for me, I have *always* fine-tuned my square root approximations using the AM-GM inequality. Kidding! I'm kidding! If I ever need a square root approximated, I have you PMP eggheads to turn to. Oh, and for the record, there was a typo in Christopher's answer to your question. He said, "*The AM-GM inequality assures us that our approximation will be greater than or equal to the exact value. In fact, equality only occurs when a = b.*" This was published as "*when a = bb*."

$$\sqrt{ab} \leq \frac{a+b}{2}$$

The Arithmetic Mean - Geometric Mean inequality (AM-GM)

$$\sqrt{ab} = \frac{a+b}{2} \iff a = b$$

Anyway, now that we've cleared that up, I wanted to ask you: how did you hear about the PMP, and how do you know Christopher? Everyone I've profiled for the newsletter up to now is/was in the same facility as Christopher, or is/was a mentor to an inmate in that facility—until you.

JW: I don't know him personally. However, I remember when I first read his story in *Popular Mechanics* and I was struck by what he has been able to accomplish. His inspiring story motivated me to write the PMP in January 2021, with the hope of finding a mentor who would give me focus and direction in my own mathematical journey. The PMP gave me the boost I needed to grow when I felt stagnant. Since the beginning of my correspondence with the PMP, I have promoted the organization to others who have expressed interest. Since then, the PMP has paired at least three other inmates here with mentors.

CF: That *Popular Mechanics* article brought a lot of people together, both inside and outside prison walls.

So, who's your mentor? And how's it going?

JW: My mentor is Professor Tim Pennings. Before learning of the PMP, my ability to gain insight into the world of mathematics was nonexistent. My limited knowledge consisted of a few algebra and calculus textbooks either mailed from family or provided by the institution's library. Through our correspondence Mr. Pennings assessed my needs, then afforded me the opportunity to study higher mathematics such as set theory, real analysis, abstract algebra, differential equations and mathematical modeling. I am truly grateful and fortunate to have his guidance.

CF: I met Dr. Pennings via Zoom at the PMP's Pi Day 2021. His presentation, entitled *"Do Dogs Know Calculus? Bifurcations at the Beach*," starring his adorable little Welsh corgi, Elvis, was enchanting, even for non-mathematicians like me. Did you know that it was the same article you read in *Popular Mechanics* that

motivated Dr. Pennings to reach out to the PMP? A bit of serendipity there...

JW: Indeed. Sadly, with no access to the internet, I haven't watched Tim's Pi Day presentation. I'll have to wait until I'm released—but I can imagine the interaction between Elvis and Tim.

CF: There's a rumour that, besides doing the running, swimming, stick-fetching and calculus computations in his head, it was actually Elvis writing all those papers Tim gets credit for... ☺

Were you always interested in math? How do you see it figuring into your future?

JW: Yes, I've always enjoyed mathematics. In high school I took geometry and advanced algebra simultaneously. My inclination toward math helped me score high enough on the military's placement exam to have my pick of military jobs. In addition, I enjoyed technology and began pursuing a BS in Computer Science. Sadly, with just three classes remaining, I am unable to complete the degree due to my incarceration. But I'm not done with college. I plan to finish this degree once I go home, then continue on to a higher degree in mathematics.

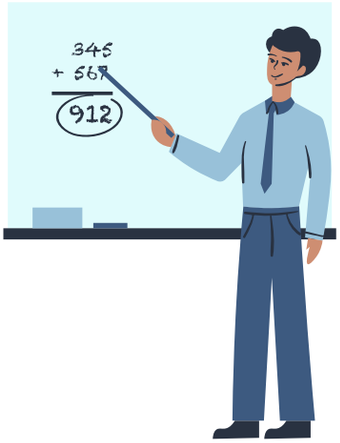
CF: Sounds like an achievable ambition. Forgive my ignorance, but what's the difference between a disciplinary barracks and any other penitentiary?

JW: The USDB at Fort Leavenworth is a military prison that houses incarcerated army personnel with sentences longer than ten years. However it also houses other branches of the military, like the one in which I belong: the air force. The great thing about being in the USDB—if there is anything great about being in prison—is that I am surrounded by other military members with similar experiences. We all relate to each other in deployments, assignments, training schools, and the feeling of brotherhood.

U.S. AIR FORCE

CF: The PMP is about inspiring change. Do you consider yourself to be different than when you first ended up behind bars? If so, to what do you attribute the change?

JW: When I arrived at the USDB, I looked for ways to put my life back in focus. I used what I knew at the time to help others with their math homework, and soon word of my tutoring spread to the academics department. I had no prior teaching experience, but the department head offered me a position to teach basic arithmetic to fellow inmates struggling to pass the Adult Basic Education test.



Over time, the program grew to what the academics department offers today. So far, the math programs offered are FAST Math, College Algebra Prep, Math History, Scientific Calculator Skills and Pre-Calculus. I developed the last three courses. Through teaching I learned to be patient, to listen, and to understand while casting out any prejudices. There is something special about teaching mathematics. I learned in my own way: through struggle and discovery. Now, when demonstrating a technique such as factoring, I try to show the student several ways to find the result. Sometimes a student will have that "Aha!" moment and suddenly they *get it*. They finally figure out how to solve the problem—and sometimes they wonder

why it seemed so difficult in the first place. This is why I enjoy teaching mathematics. It's very rewarding.

CF: I wonder which is more rewarding, being the student who all of a sudden sees the light, or being the teacher who threw the switch? I think I know the answer in your case. What are your goals for when you are released, Jesse? And when is that?

JW: My long-term goal, after I am released in July 2023, is to pursue a degree in mathematics, eventually earning a Ph.D. I don't know the precise date of my release because, as I continue to work, I earn abatement days. However, at this point, I'm not certain where I should concentrate my efforts. The field is very broad.

CF: I can only imagine. Any inclinations one way or the other at this point?

JW: I'm leaning toward applied mathematics. One day, I would like to contribute to the mathematics community in a meaningful way, like publishing research.

CF: Don't forget to add mentoring another PMP participant!

What are your other interests?

JW: Every inmate is required to have a job in the USDB. I work in the wood shop. It's a full-time job where I create building plans and draw custom projects using AutoCAD. After 4000 hours on the floor milling lumber and assembling projects, I earned my apprenticeship as a wood machinist. When I go home, I would like to use this skill to start a business repairing and restoring old furniture.

CF: That would certainly be a rewarding business. It sounds like you're actually pretty busy there.

JW: Yes, I do keep myself surprisingly busy. In addition to my full-time job in the wood shop, I have managed to take several college courses through the local community college. I was once elected as the education director for a cultural diversity group, and I am the current editor of the facility's newsletter, "*The Passing Times*."

CF: So we both have "newsletter editor" on our resumes.

JW: We do. And of course, I teach and tutor other inmates in mathematics, as I've mentioned. Really, I have an amazing gig here. Who would have thought being in a controlled environment would afford me so many opportunities? Sometimes I wonder if I'll get the same opportunities when I go home.

CF: I guess we'll see, since you have less than a year remaining on your sentence. But it sounds like you've made good use of your time in prison. A wise (incarcerated) individual once told me that prison can either be a womb or a tomb. The PMP is one way for the mathematically inclined to chose the former.

JW: I agree. I'm fortunate to have had a great number of opportunities in the USDB. I have met many other inmates and seen them for the complex people they are. I got to know Chelsea Manning, whose sentence was commuted by Obama, and Clint Lorance, who was pardoned by Trump. Everyone's journey is different, but at least I get to meet people and be a part of their journey to success.

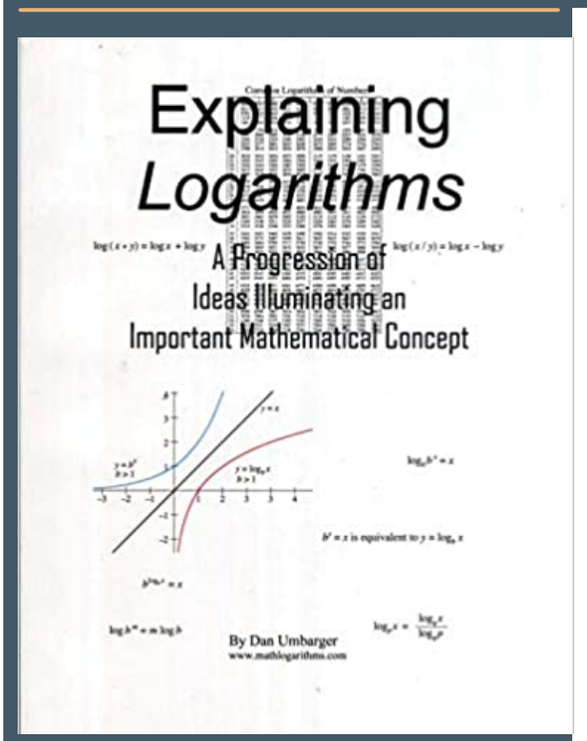
JW: I have an inspiring story I'd like to share. A death row inmate's sentence was reduced to a life sentence through clemency. The facility moved him from special housing to general population just in time for the next iteration of the Algebra Prep class I was about to teach. He eagerly signed up. I knew he had just come off death row after spending over 20 years there, so I was very perceptive to his progress in the class. He was one of my best students—he didn't miss a day and completed all the homework. He even found some errors in my worksheets! And he had the best attitude I've ever seen in someone in prison. He told me that the first day he got to go outside to our recreation yard, he knelt down and kissed the grass. To this day, he still has a big smile on his face each time I see him.

CF: Talk about a second chance! That *is* inspiring. Anything else you'd like to share for our newsletter audience, Jesse?

JW: Just that, above all, I'm grateful to be of help to others. I hope I can bring my skills and my desire to help people to the outside world and they will accept that I am genuinely trying to help. And I thank you for this wonderful opportunity, Ms. Finlayson.

CF: You're welcome, Jesse. Remember, the PMP likes to stay connected with people post-release. We hope to hear good things about you in the future.

BOOK REVIEW WINNER



Our winning review of *Explaining Logarithms* by Dan Umbarger makes for compelling reading for math people and non math people alike. We've published the full review on the next page.

The **winner** of the book review contest is...

PAUL MORTON, WOODBOURNE, NY

At first glance, *Explaining Logarithms* seems like a typical subject-oriented workbook, but by the time I had finished reading the forward, I knew it was much more than that. The author's key emphasis from the beginning is intuition: understanding the math.

Being a self-taught "math person" (I don't use the word mathematician lightly), my biggest struggles involved not being able to grasp the intuitive concepts quickly in certain subjects of math because of the way the textbook was written, spending countless hours reading and rereading the text and searching for other resources. Mr. Umbarger's approach in this book is much different than the usual "proofy" textbook approach. In the very beginning, he explains how and why logs and antilogs were developed. As the book progresses, he explains how they evolved from their initial use to how they are used today. I was surprised to learn that there are many more scenarios in which to apply them then I had originally thought.

Also, Mr. Umbarger explains all of the mathematical rules associated with logs and antilogs by using examples to demonstrate these rules in a very easy to understand, step-by-step process that kept me highly engaged. Also, there are many high-quality graphs and visual aids in this book to help the reader understand intuitively how log and antilog functions work.

I was also impressed by Mr. Umbarger's ability to convey the elegant relationship between the natural world and the number "*e*." He explains some concepts about "*e*" and the natural logarithm that most people do not get to learn until calculus. In my opinion, many of today's textbooks are lacking in intuition-driven mathematics. They take the math out of mathematics and replace it with instructions on how to use your graphing device. Mr. Umbarger puts the math back into mathematics and is able to give the reader a clearer and more intuitive understanding of logarithms and antilogarithms than any other textbook I have read. Great job!

A must-have for anybody studying college algebra or precalculus.
And here's a "You've really got to get this book!" for math lovers like me.

Christopher Havens

A MISCREANT'S MISCELLANY

by Christopher Havens

Christopher Havens in Conversation with Tian An Wong

Tian An Wong

CH: Greetings math lovers! How awesome is that feeling you get, after exploring a topic in your studies, when a universal fact is laid bare ... when you achieve an understanding of something that was not explained in any of your books or material? The joy and fulfillment of mathematical discovery is a truly beautiful thing, regardless of whether the discovery was brand new or not! Either you've discovered some previously unknown nuance in the language of the universe or you're sharing the same idea with another mathematical mind from some other time and place. In any case, whether it be the general idea for adding two rational numbers or a number theoretic theorem, you have just experienced one of the feelings that lead so many of us to pursue a mathematical lifestyle.

Today we have with us a wonderful mathematician, Tian An Wong. One of An's gifts is in guiding people toward the joys of mathematical discovery! An, why don't you tell us a little about yourself?

TA: Hello! I'm so excited to be talking with you! I'm Tian An, or An for short—my first name has two words. It's a Chinese name. I'm originally from Malaysia, a country in Southeast Asia right below Thailand. I came to the US to attend college, where I fell in love with math. It's a funny story: I was bored working in the library one summer, and I was wondering about how the universe came to be. So I thought about the big bang and figured maybe I should learn some quantum physics. But when I opened a book on quantum physics, the first chapter was all math! I didn't understand any of it, not having taken any math since high school. So I signed up for some math classes, then some more, and then some more, and here I am! I never did go back to physics!

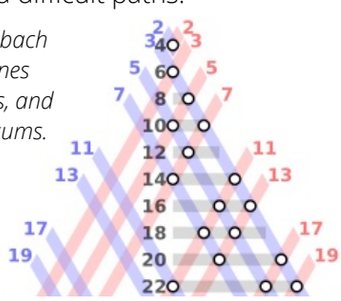
CH: I love it! I once had a huge interest in astronomy and cosmology; this was right when I first started studying math. I decided that I had better get to working on the foundations, but like you, I fell in love with math—so much that I didn't want to do anything else! I think we all owe a big thank you to quantum physics and cosmology, because if it weren't for those subjects, we might have only half as many mathematicians!

I think most of us who have mathematics in our hearts have at least one area that we can get completely lost in, every single time. I mean, lost to the point where it's hard to maintain social interactions until we finally re-emerge from the rabbit hole. What area of mathematics grips you the most, An? It's often a niche, right?

TA: Yes, I like to think that every mathematician also has something else they love and sometimes are

very talented at. It's often quite "unscientific," like music or juggling or poetry. What turned me on to mathematics, and really still has a lot to do with my current work, is number theory. Number theory is the study of prime numbers—like 2, 3, 5, 7, 11, and so on—that can only be divided by themselves and one. That there are infinitely many of them is a theorem that is over 2000 years old! But there is still so much we do not know about prime numbers! People are still discovering new things about them and new things to do with them every day!

The best example is the Goldbach conjecture from 1742, which says every even integer can be written as a sum of two prime numbers, so like 4 = 2 + 2, 6 = 3 + 3, 8 = 3 + 5, and so on. If you don't believe it, just keep going and see! This has been verified by computers to over one million million million (that's 18 zeroes!), but that still doesn't PROVE that there isn't *beyond* that some number that will fail the conjecture. So there are things that computers just cannot do. They give us every reason to *expect* the conjecture to be true, but that's not the same as a *proof*. And of course, mathematicians have tried and failed for centuries! It's problems like these that grip me the most. They are so simple to state but lead down really deep and difficult paths.



One visualization of the Goldbach Conjecture. Red and blue lines correspond to prime numbers, and their intersections represent sums.

The nodes on gray lines give a possible representation of that even number as a sum of two primes

CH: I know what you mean there, An! One of my biggest goals in research is to solve the problem of predicting the period length of the continued fraction expansion of √(*N*), in terms of *N*. Finding tricks to determine the period lengths of families of these numbers isn't too difficult, but a general formula for all *N* is so far unknown. Solving it would be the Holy Grail for me.

What would you say to the explorers among our readers who have never approached an open problem in mathematics? I mean, are there accessible problems out there for the amateur which have *not* been solved?

TA: That's a great question, Christopher! While it certainly is true that number theory holds some of the deepest and oldest unsolved problems, some of which finally yield to sophisticated state of the art technology, there are tons of problems that are completely accessible and just waiting for someone to take a hard look! Just as number theory has outstanding open problems that are easy to state and yet unsolved after hundreds of years, there are also number theory problems that are easy to understand and simple enough to work out with just a bit of hard work and stubbornness. Math research is about not giving up. It's like doing a hard crossword or sudoku puzzle, but you just keep coming back to it day after day, making little bits of progress and mistakes along the way until it is finally solved. Over the years, I have guided college students who have just a bit of math background in number theory and research projects, and in many cases our work has ended up being published in a peer-reviewed academic journal!

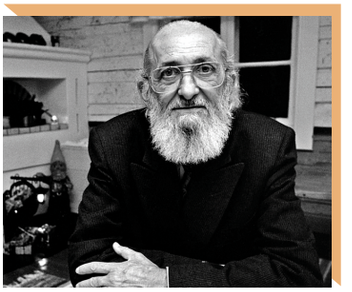
5	3			7				
6			1	9	5			
	9	8					6	
8				6				3
4			8		3			1
7				2				6
	6					2	8	
			4	1	9			5
				8			7	9

But there is one catch for someone with a limited math background: These open, accessible problems are hard to find on your own. When you first set out trying to do math research, it is hard to get an idea of the lay of the land by yourself—what has already been proven and what has not. This is where having a research guide from the PMP comes in!

CH: So correct me if I'm wrong, but this sounds a lot like a professor guiding their students in

undergraduate research, except instead of students, we're talking PMP participants. Am I on the right track? This sounds cool! Can you tell us what a research guide does?

TA: In one sense, yes, it's like a professor guiding undergraduate students in research, in the sense that one leans heavily on the professor's knowledge of which problems are open, interesting, and approachable. It takes some skill and experience in coming up with problems that fit all three criteria—though once you get started on a math problem, one of the questions tends to lead to another, and before you know it you've got a lot more problems than you have time to solve! On the other hand, there are important ways in which it is different: Whereas undergraduate students tend to be teenagers, PMP participants are usually older and often have life experience that is richer than that of your average professor! As Brazilian radical educator Paolo Freire described it, students are student-teachers and teachers are teacher-students. We all have much to teach and learn from each other, and that rich interaction is one of the beauties of the PMP.



Radical educator Paolo Freire (right) wrote the famous book Pedagogy of the Oppressed

CH: Well said! I feel that researches such as this are one of the purest ways in which we can serve justice after our past mistakes and where we find meaning and beauty that transcend the razor wire surrounding our minds. There really is no better way to achieve freedom before walking out of the gates of prison than engaging in work that *truly* makes a difference in the world, even if it's only one theorem at a time.

I imagine a few of our participants may have the same questions that I currently have, An. How does one know if they're ready for such problems? Also, many of our participants may not know what it means to work open problems. How would you describe this type of work in terms of contributions to society?

TA: These are all hard questions! To start with, I would say that a minimum requirement for working on open problems is this very subjective thing we call "mathematical maturity." This involves knowing what counts as a "proof" of a mathematical statement, having the stubbornness to not give up easily, but also the creativity to continuously try new and different methods, being able to judge for yourself what is mathematically right or wrong, especially when it comes to your own work! This takes a lot of practice, reading and doing math, and it's much more efficient when you have a guide like a mentor.

As for making a difference in society, I think you are better positioned to answer that question than I am. In my view, to simply be able to work on mathematics within the confines of a prison can be a politically radical act: The US prison system is about breaking the spirit, dehumanizing people and exploiting incarcerated persons for their labor. The study of mathematics has a lot of practical applications, of course—in engineering and science and so on, but for many of us, we do it simply because we like it, for the beauty of it. So to be able to carry out an intellectual pursuit, to produce new and beautiful mathematics within the prison system is an act of creative defiance. It is not so much about being productive as it is about being human despite all attempts to dehumanize you. And that is radical.

What's your take on it, Christopher?

CH: Well, my thoughts on this are going to be a little different than you might expect, being that I'm in prison. For me, doing mathematics is an endeavor of the heart, and the adversity I face while studying and researching in this environment comes, unfortunately, more from the people around me than from the actual system itself. I will agree that the prison system is not designed for people who are making a difference in the world and who are making meaningful contributions to society. The "anomalies" who work towards making real changes and contributions in this setting are often seen as having some angle. But there's room for optimism. Many of the prison systems that house our productive PMP participants have superintendents who truly want to help them succeed, so while it *is* the job of the PMP to help our participants navigate the adversities of the prison environment, it is also our job to build trust among superintendents and other administrators at the prisons so that we may educate them in how we can identify and better serve prisoners who are the anomalies: the ones who are transcending their pasts and working towards justice through **real* change, **real* contributions and **real* community involvement through the PMP.

Studying mathematics with any degree of seriousness in this environment is an uncommon act, and perhaps even a bit radical, as you say. It takes grit and a drive to progress even with all of the external noise and distractions that prisoners experience. When I research mathematics in prison, it's always something that no other prisoners understand. So we who endeavor in this practice do so without any satisfaction other than the healing it brings to our hearts through joy and beauty. We will never get "kudos" from our peers. But although our work behind these walls is often unnoticed, researches such as the ones guided by An mean that our time is spent in an effort to contribute to the wealth of human knowledge, if even in a small way. There is no greater feeling or honor than knowing that you're working towards **justice** through contributions to mathematics.

TA: I don't disagree! The deeper you go in mathematics, the lonelier it can be until you find your people, and I hope that through PMP, individuals who want to pursue math can get connected to like-minded people! Here at PMP Research, we are just getting started, working with a handful of folks who are interested in or are already doing mathematics research in prison. Christopher, what advice do you have for readers who are interested in getting a taste of research? What about those who might be interested, but aren't sure if they're ready?

CH: My advice is this: If you're already engaging in your own research, then there are countless things you'll need in publishing new math to a professional journal, like formatting in LaTeX and navigating the submission process with the back-and-forth between referees. You'll need guidance, encouragement and real critique without sugar-coating. We can provide journal articles and pretty much anything you can think of. We want to be your supporting institution through all of your research endeavors.

On the other hand, if you've never researched but you're not afraid of formal mathematical proofs and engaging in problems that can take month (even years), then this can be one of the most meaningful and humbling experiences of your entire life. And if you're not sure if you're ready, I would suggest

finding a book in number theory or a book on mathematical proof. If you find that the material is to your liking and that you can bend your mind into outside-the-box alternate proofs, then it can't hurt to dive into the "rabbit hole."

To get started, you would write to us the same as if you're writing to your mentor. The only difference is that you'll need to address the letter or the JPay message to Tian An. To do this, just add the following at the top of your message:

PMP Research - Tian An Wong

That'll get you started. If you're already researching, make sure to let An know what your needs are. If you've never researched, then make sure you convey this. We may send you some "test problems" to gauge where you're at!

We hope to see you become involved with work that is truly an endeavor of the heart. An, thank you so much for a most wonderful conversation! Thank you again for giving such beautiful opportunities to this truly underrepresented population.

INTRODUCING...

HUMANS OF SAN QUENTIN

Are you a writer? Is there a story in you just burning to be told? HUMANS of San Quentin *can help make it happen. Our Chief Executive Troublemaker recently made the acquaintance of Diane and Ashley, two wonderful women from the* HUMANS *team. Like the PMP, this organization is devoted to human flourishing, and they do it by collecting real first-person stories told by people currently incarcerated in all prisons. They feature the voices of these people on their website and/or social media channels. Their efforts are helping to change the public's perception of humans behind bars by reminding us of our shared humanity.*

Diane Kahn
Founder: Humans of San Quentin

We thought we should be good neighbors and introduce you to HoSQ. In return, they will be sure to direct any math nerds our way, but no strings attached, honest! Like-minded organizations are happy just to lift one another up. ☺

Hi from Humans of San Quentin!

We are a humanitarian nonprofit out of San Quentin State Prison aiming to amplify the voices of incarcerated individuals across the globe. We want your voice to be heard by the outside world.

My name is Diane Kahn, and I am a volunteer inside San Quentin on a mission to collect real stories from incarcerated people and share them with the world

to demystify stigmas, raise awareness, and cultivate community. While instructing English for San Quentin's GED program, I was struck by the palpable vulnerability and emotional intelligence my students communicated, and I knew their narratives needed to be shared beyond bars. Along with the help of co-founder/incarcerated journalist Juan Haines and the inspiration of a photojournalism project in New York by photographer Brandon Stanton, HoSQ was born.

Stanton's series, *Humans of New York*, features small vignettes from everyday people walking the streets of New York City. Similarly to the photographer's desire to make people feel less like strangers, we strive to highlight the individual stories of the incarcerated to humanize them beyond identification numbers. At first I only dreamed of running a project like Stanton's, but with the aid of Juan and numerous devoted team members, this dream is now a reality. Our relentlessly hard-working team is a fusion of volunteers, interns and employees, many of whom are currently or formerly incarcerated.

Since our inception in 2020, we have expanded to reach people imprisoned in all 50 states, and more recently, in international prisons. Together we are devoted advocates for social justice, aiming to build compassion and solidarity through storytelling. Our end game is to build a collective consciousness surrounding incarceration and rehabilitation; by shining a light into every cell, in every part of the world, we hope to change the narrative.

We want to hear from you! Share your story (written in first person) and photo(s) of yourself. Be sure to NOT include facts about your trial. Include your writing, birth date, years incarcerated and any pictures to:

P.O. Box 417
San Quentin, CA 94964

(415) 295-7139

hi@humansofsanquentin.com

If you're in Canada, please have a friend or loved one reach out to us via email.

Submission Guidelines:

- Your full name
- Your age/birthdate
- At least one photo of you inside - but the more photos the better, from any point in your life. We can post up to ten!
- Years incarcerated
- Your mailing address
- Can we send you stamps/envelopes?
- Is there a limit to the number of pages we can send you?
- Can we help you get pictures from someone outside? You can share their contact info and we will obtain pictures for you.

We are *not* looking for opinion pieces; we want to hear about *you* and *your life*. Keep it personal and keep it fairly short.

Writing Ideas:

- Share a conversation you've had with someone.
- Dive into a relationship you have/had.
- What have you learned about yourself in prison?
- What would your family be surprised to know about you?
- How do you communicate with your friends and family?
- What do you miss most about being outside?
- How do you see love?
- Share a childhood memory.
- What gets you through each day?
- What brings you comfort?
- What or who led you to prison?
- What keeps you up at night?

If you choose to submit a piece, you will hear from us to acknowledge that we've received your writing. (We will return any pictures you send us.) If you are featured, we will send you a copy of your post from our Instagram feed, our website or other social channels. It can take two or three months before you are featured.

CLOSING COMMENTS

"We are story. All of us ...you, me, us, together. When we take the time to share those stories with each other, we get bigger inside, we see each other, we recognize our kinship – we change the world, one story at a time..."

-Richard Wagamese (1955 – 2017)

Claire Finlayson

The truth of those words can by illustrated by what happened after a certain story appeared in the Feb 21, 2021 issue of Popular Mechanics. It was an article by A. C. Shilton titled *This Inmate used Solitary Confinement to Learn Math*. *Now He's Solving the World's Hardest Equations.*

Reading that story was the impetus for a then-15-year-old Walker Blackwell to contact Christopher Havens to offer help catapulting the PMP over the prison walls and into the international spotlight. The same story pulled in mathematically inclined inmates like those featured in this iteration of the newsletter: Paul Morton, for whom a love of mathematics has become foundational, and Jesse Waite, our featured participant. It also drew in Tim Pennings, the mathematician who became Jesse's mentor.

The power of Christopher's story is the reason I found myself drawn inexorably into the PMP's orbit, albeit without a breath of mathematical aptitude. (If you think I'm just being modest, go back a few pages and read my pathetic plea to the Math Guru...) But in reaching out to PMP participants for the Spotlight column, I have seen how a shared passion for math can create community. I have seen fundamental paradigm shifts in people whose passion for mathematics has been awakened or reignited and nurtured. I have heard the word "joy." I have seen the power of human connection when a mentor and mentee "click."

Christopher Havens has a network of friends that reaches around the world, most of whom he has never met in person, and from among these he has managed to assemble an impressive team of volunteers to tend to all the moving parts of this expanding organization. This he has accomplished under, shall we say, less than ideal circumstances. There have been days so discouraging he could have given up trying to help other incarcerated individuals connect with the math community and retreated into research where he's happiest, or ride the wave of his own celebrity and forget about everybody else, but he has persevered, and now the PMP has its own forward momentum. And it will keep changing lives as the story spreads.

If you're on the outside, consider helping out financially with a one-time or monthly donation.

https://www.prisonmathproject.org/donate-supporters/

Your support will allow us to send math books to all inmates who request them, and it funds initiatives like this newsletter.

If you're incarcerated, consider participating in our upcoming book review contest—in our next iteration we will be featuring the inspiring title, *Mathematics for Human Flourishing* by Francis Su. Try your hand at the Math Warden's logic puzzles. I'm always looking for fascinating people to profile, and I'd love the perspective of a mentor next – reach out to me if you would like to share your experience with our readers. (See the Participant Spotlight column for my contact info.) Or simply drop us a line and let us know where you're at on your math journey, whether it's brushing up on your times tables or trying your hand at original research.

And by the way, you don't have to be a "genius" or a math prodigy to be paired with a mentor.

We have experienced some communication glitches, but we now have a specific team tasked with addressing mail and communications between mentors and mentees. Let us know if you're still having problems, and we will do our best to help solve them. Do you have ideas to make the current system better? We're listening.

We greatly value your feedback, and we want to keep in touch after your release, where, in some ways, the "real work" begins. So please give us a heads up as to your release date.

To close, I quote Francis Su:

> *"There truly is a need, even among those who do math for a living, to talk about our longings for the common good, and the need for us to be better human beings to one another."*

Wishing you all the best until the next iteration.

Claire Finlayson - Newsletter Editor and PMP Superfan

PMP
PRISON MATHEMATICS PROJECT

PMP

PRISON MATHEMATICS PROJECT

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