



PMP

PRISON MATHEMATICS
PROJECT

THE PRISON MATHEMATICS PROJECT NEWSLETTER

ITERATION 4

IN THIS ITERATION

- 2** Ask the Math Guru
- 3** Insider Insights
- 4** Participant Spotlight
- 7** The Prisoner's Dilemma
- 13** Cool Math from Another Time and Place
- 16** Book Review Review
- 20** A Miscreant's Miscellany
- 25** Closing Comments

Welcome to Iteration #4 of the Prison Mathematics Project newsletter! We celebrate inclusivity and diversity in the math community by reaching lovers of mathematics, whether incarcerated or not.

Our writers, editors and contributors from around the globe have designed this newsletter to share with you our deep appreciation for the wonderful world that can be discovered through the study and exploration of mathematics. We connect prisoners who are dedicated to change to a community that provides an essential framework for rebuilding their lives during their incarceration. The aim of the Prison Mathematics Project is to provide opportunities for our participants to experience a new lifestyle, a new culture that leads to human flourishing through the transformative power of a passion for mathematics.

ASK THE MATH GURU

Do numbers freak you out? Does math send you into meltdown-mode? Are you wondering why we need to learn this stuff ANYways?? Don't worry, The Math Guru is here to help you work through your math trauma, one problem at a time. Ask for advice, guidance, or just a good ol' pep talk! You got this!



VaneSSa Vakharia,
The Math Guru

Hello Vanessa Vakharia,

I recently received the PMP newsletter and thought to myself "Wow, what a brilliant idea!" This is such a cool way to update your members and get everyone on the same page.

I am most excited about the "Ask the Math Guru" section. I feel many a time we have questions that we don't understand how to phrase or just didn't think to ask! With this section, I believe we would be able to view problems from another's perspective, shedding light that guides us on our math path.

I look expectantly forward to receiving the next issue of the PMP newsletter. Please keep up the excellent work!

God Bless,
Rey A.

Rey!

Thank you so much for reaching out. I love what you said about the purpose of asking questions. It's so true that when we ask questions, we often inspire questions in others - and even help others realize that they HAVE burning questions in the first place. Asking for help is a sign of courage and curiosity, and I am so so excited about all of the questions I have been getting from all of you. While this column isn't focused on questions involving math content, any mathematical questions sent my way will be answered by others! In this section, I hope to provide guidance on how to enjoy your math journey, how to deal with obstacles, and anything else that you might need support with. Math is a journey, not a destination, and I'm pumped that you're along for the ride!



VaneSSa Vakharia

Got math anxiety? Think you're a hopeless case? VaneSSa to the rescue! The Lady Gaga of mathematics will put her Master's in Mathematics Education to good use by delivering the ultimate personalized pep-talk!

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 The logo for 'INSIDER INSIGHTS' features the word 'INSIDER' in blue, bold, uppercase letters above the word 'INSIGHTS' in the same style. To the left of the text is a large, stylized black square root symbol.

In this column, we publish feedback from PMP participants, mentors and volunteers. Do you have something to say that will educate, encourage, entice or entertain your fellow math students? Drop us a line at the address on the back page, or if you have access to email, send a message to PMP@pmathp.org. We welcome your input.

We received this letter from participant John Baran.

I am writing you today to explain what the Prison Mathematics Project means to me.

I am a 36-year-old autistic male confined in a Washington State prison. I am not talking about what I see myself as, but what those around me think of me. To them I am just Prisoner Number 353800.

From the age of seven I was in the foster care system. I moved through 31 foster homes, mental institutions and group homes. There was no stability in my life. Then I found math: a safe and constant thing for me to understand. When I came to prison six years ago, I had math to keep me company, but it was simple math, because without my calculus books and other texts I had no way of continuing my math education, so it began to slip out of my head. I lost my ability to do advanced calculus. I have been put in one mental health unit after another because of my autism, and this has made it hard for me to talk to anyone about advanced mathematics, so I spend my time working with people to get the math part of their GED done. I have written this explanation of where I was before the Prison Mathematics Project so you can see how lost and alone I felt.

When I heard about the Prison Mathematics Project I was very skeptical. I have tried to get in contact with other education places before.

The first thing they ask for is a check from me and a self-addressed, stamped envelope. Then they say they will contact me in 6 to 12 weeks... So when I found out about the PMP I didn't have much hope, but I sent a request on J-pay and got a response back within 24 hours. When that response included a request for my address so they could send me an application I started to feel very hopeful. This was because I had found a place where people thought about math like I did. My hope grew stronger when I got my first letter from my mentors, Robin and Ale.

← hope →

If you asked me what the Prison Mathematics Project means to me I would tell you *hope*. Hope that I can make something of myself. Hope that I am not a lost cause. Hope that with help and understanding, through something that I love, I will never set foot in a prison again. I have hope. This is what the Prison Mathematics Project means to me.



John Baran



Walker Blackwell

PARTICIPANT SPOTLIGHT

by Claire Finlayson

“

Well, someone has to keep these math geeks in line and jump on every little spelling mistake...

I'm a writer from BC, Canada, so don't try to bust me for using British spelling, like "cheque" and "colour," okay? I am a huge fan of the PMP in spite of my rudimentary math skills, so how could I help? Well, I'm a storyteller at heart and nonfiction is my jam. It's the unique true-life stories of people that interest me. So I write profiles of participants and volunteers and others involved in this program.

”

If there's someone you'd like me to spotlight, here's how you can contact me:

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Claire Finlayson

I have the pleasure of interviewing Walker Blackwell, co-founder with Christopher Havens of the Prison Mathematics Project. Walker is also the content manager for this newsletter. It's his job to liaise with our contributors and make sure all their columns are ready before we go to print. This will be his first iteration at the helm, so if there are any glitches, you can blame him.

CF: Walker, hi. We've had a lot to do with each other since I conscripted you to help with the newsletter in the spare time you don't have. But let's backtrack a bit. I first heard you on a radio podcast here in Canada when you and Christopher were interviewed by Matt Galloway, host of CBC's *The Current*. In that podcast you explained the genesis of your involvement with the PMP, so I'm somewhat familiar with your story, but for the benefit of our readers who don't know you, can you briefly retell it?

WB: Absolutely! In May of 2020 I read an article published by Marta Cerruti in *The Conversation*. This article detailed Christopher's accomplishment in mathematics and mentioned a math organization he

had started in prison. His story inspired me, so I contacted him asking if he wanted to partner up and expand his math organization nationwide. He seemed really enthused about the idea. For the next few months we sent endless emails to each other, planning the PMP's future. Slowly it started to evolve into what it has become today.

CF: I see. So the PMP was Christopher's brainchild, but you're helping him take it to the next level. You have so many interests. Whatever drew you to this particular project? And did you encounter any roadblocks on your way to getting involved?

WB: I've always been a bit of a math nerd myself. Once I even dressed up as an imaginary number for Halloween!

CF: Umm, you're gonna have to explain that one, Walker.



WB: Well, we went to Party City and bought ghost cloth, and I wrapped it around myself. Then I stuck numbers on it and the transformation was complete. I thought it was pretty funny! My love for math was definitely a driving force in my desire to get involved with the PMP. Also, my curiosity compelled me. I didn't have much knowledge of the prison system when I first decided to reach out to Christopher, but I was eager to know more!

CF: Okay, it probably bugs you when people make a big deal about your age, but I'm going to do exactly

that. Please tell us how old you were when you got involved with the PMP, how old you are now, and what you're up to that keeps you so incredibly busy. Also, what are your future goals?

WB: Haha! I reached out to Christopher a few days after I turned 15, and I'm 17 now. Most often I try not to mention my age when in conversation with other members of the organization because I don't want to receive any sort of special treatment. With that said though, it is often quite funny the responses I get when people find out they're talking to a teenager.



Outside of the Prison Mathematics Project I am heavily involved in speech and theatre. I have been acting since I was in first grade and am a huge theatre fanatic. I will come to a crossroads when choosing my future career path. While I would love to do something with math and maybe even finance, I also love performing! I would love to work in the film industry!

CF: You are a live wire all right, kid. It really does seem like you could turn your hand to just about anything in life and succeed at it. What is your current role in the PMP? And what are your hopes for the future of the organization? I hear it's expanding to all 50 states and into Canada. That's ambitious, given that the whole thing is run on a shoestring budget and almost exclusively on volunteer labour.

WB: Currently I am Newsletter Content Director, as you mentioned. You and I are in charge of handling

this great segment of the organization! All inmate participants receive a mailed copy, and of course there's the online version for people with access to the internet. As the Prison Mathematics Project expands I hope to increase our sphere of influence. Since our program is the first of its kind (in regard to reducing recidivism through mathematics) we have a unique opportunity to develop research on the effectiveness of this method and measure the personal growth of our participants. I hope our findings will spark other efforts to increase educational opportunities in the prison system.

CF: What would you say you've learned? Or maybe a better question is: What have you valued the most from your association with the PMP?

WB: My biggest takeaway from working with the Prison Mathematics Project is the insight I've gained regarding the prison system. Our schools are a bit spooked when it comes to talking with students about the prison community, but I think it's important that society learns to open dialogue on the subject. This experience has opened my eyes to many of the issues prisoners face, but these issues aren't being publicized because the topic is almost taboo in social settings. I think we have a ways to go in changing this. Due to the scarcity of conversations surrounding this facet of society, many people gain their impressions of prisoners solely through Hollywood productions. However, if this organization has taught me anything, it is that those stereotypical characters do not represent the prison community. I'm glad I took it upon myself to understand the culture of the prison system. I think there is a big misconception that prisoners are far off, mysterious entities, when in reality they are just like you and me.

CF: You've made an important point, Walker. For our last newsletter I interviewed Rory Andes, an inmate at Monroe Correctional in Washington State. He said in a hand-written letter to me, "I spent the majority of my life seeing crime in black and white,

but that's so untrue. People are complex, even in here." So even convicted criminals facing incarceration have this stereotypical, clichéd view of who and what inmates are—until they are inside and see that there are *all* types, just like on the outside.



The main thrust of the PMP is, of course, mentorship for inmates who are already engaged in math studies. But it's about so much more than numeracy! Being paired with a mentor—a mathematical friend who is interested in an incarcerated person's future—has been shown to be a powerful antidote to recidivism. So I applaud your activism and your passion, Walker. What would you say to others who would like to contribute in some way to the mission and vision of the PMP? Do all potential volunteers have to be mathematical geniuses?

WB: Regardless of who you are, if you would like to help out with this organization, LET'S DO IT! Many of our volunteers join as mentors and pair with one of our program's participants to help them in their mathematical journey. But volunteers can help in many other capacities! Due to our organization's rapid expansion and volunteer-based system, monthly donations are crucial to our ability to function. If you are able, this would be the best way to help us continue in our mission. If that isn't an option, spreading the word and publicizing our organization would go a long way toward ensuring the PMP's longevity.

CF: Well said! You and I are only two of a small army of volunteers. I know how hard our mentors work—our columnists and admin people, too, and everyone's working around their "real" jobs and responsibilities.

Finally, Walker, before you zoom off to debating club or acting class or your high school calculus exam—or whatever else you're up to—do you have anything you'd like to say to other kids looking to step out of their comfort zone and do something impactful in the world?

WB: JUST ASK! This has been one of the biggest takeaways for me through this organization. When I

first contacted Christopher about expanding PMP, I wasn't even sure I'd receive a response. And never in a million years would I have imagined the PMP could achieve so much in such a short time. The boldest question I've ever asked was in my email to Christopher Havens in May of 2020, and it has borne the most fruit! The worst anyone can do is say "no." And all that means is ASK AGAIN!

THE PRISONER'S DILEMMA

From the Problem Warden

I am delighted to become part of the Prison Mathematics Project through The Prisoner's Dilemma, as its (honorary) "Problem Warden." My love of mathematics – and especially sharing the joy it can bring – have been a part of almost everything I've done as a professor, financial analyst, parent, founder of the National Museum of Mathematics, author of the Studio Infinity blog, and the most recent ex-editor of The Playground (the problem column of Math Horizons, a magazine chronicling the world of math for the Mathematical Association of America). I look forward to seeing all of the new methods and creative questions that you come up with as we face the many dilemmas to come, together. And don't worry, as a one-time specialist in mathematical logic, this Warden will keep a sharp eye out for any infractions in your reasoning!

– Glen Whitney



Glen Whitney, AKA
The Problem Warden



In this iteration, we want to highlight our problem contributors. First we have Ian Stewart, whose second Dilemma appears below. Ian is an Emeritus Professor in the Mathematics Department at Warwick University. He is best known for his popular science writing, mainly on mathematical themes, for which he has won numerous awards including one of the "24 Best Apps of 2014" for his iPad interactive book *Incredible Numbers*. Overall, he has published more than 120 books including the number one popular science bestseller *17 Equations That Changed the World*. Above all, we're absolutely delighted to have him as part of *The Prisoner's Dilemma* community.

And before we get to Ian's latest Dilemma, don't forget – we're looking for solutions from you, the readers, and for proposals for new Dilemmas to appear here and online at prisonersdilemma.org. Full submission guidelines are at the end of the column.

D5: Triple Fever

Contributed by Ian Stewart, University of Warwick

Caught up in the excitement of the Derby and the Preakness, a buddy of yours heads to the track, and discovers that today is the running of the Medium-Rare Stakes. Only three horses have been entered: Tee Bone, New York Strip, and Rib Eye. Your buddy then notices the posted odds: Tee Bone is paying 4-1, New York Strip is at 3-1, and Rib Eye at 2-1.

"Hmm," thinks your friend, "that sounds too good to be true!" Knowing your math prowess, your pal manages to get you on the phone and tells you the situation, wrapping up with "And I only have \$94 on me and no time to get more before the windows close. I want to make sure I get this right. Tell me how I should place my wagers!"

Assuming your friend can only buy tickets to win, but can back any or all of the entrants with any amount as long as the bets total \$94 or less, how should the money be placed to maximize the winnings *in the worst-case scenario*? In other words, what bets will *guarantee* the most winnings, *regardless* of which horse wins the race? (And how much money would your buddy end up making?)

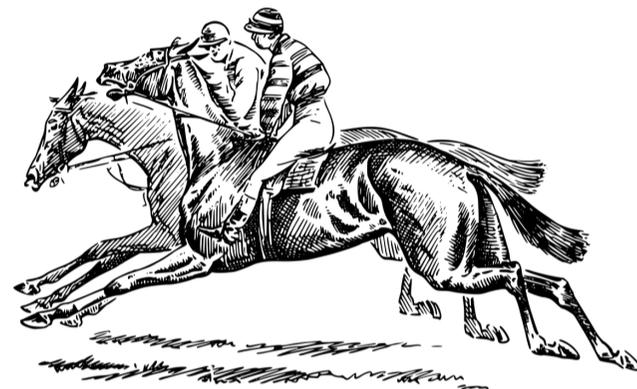


Figure 1: Race horses and jockeys

Our other new Dilemma in this issue comes from Arsalan Wares, a Professor in the Mathematics Department of Valdosta State University in Georgia. Arsalan has been a frequent contributor to *The Playground* column in the *Math Horizons* magazine of the Mathematical Association of America. He's developed quite a reputation for posing geometric problems that are both visually beautiful and mathematically elegant, and his first submission to *The Prisoner's Dilemma* is no exception. However, beware – Prof. Wares does not limit himself to geometry, as you will see in an upcoming iteration.

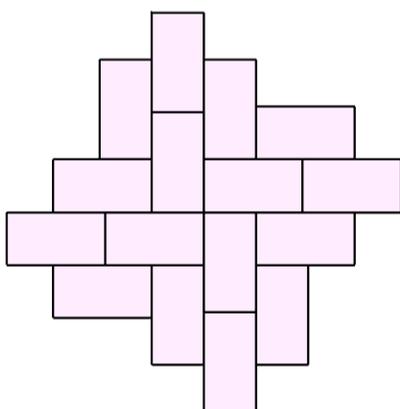


Figure 2: A pinwheel formed from 16 rectangles

D6: Rectangle Pinwheel

Contributed by Arsalan Wares, Valdosta State University

The outermost polygon shown in **Figure 2** as the region shaded in pink consists of 16 congruent rectangles placed so that their edges align (and vertices of four of the rectangles coincide at the center of the polygon). We call this polygon a *28-gon* because it has 28 edges: the line segments that make up its boundary. The perimeter of each rectangle is 26 cm and the perimeter of the 28-gon is 136 cm. What is the area of the 28-gon?

D2: Find Yourself

First appeared Fall 2021; this was a “classic” dilemma that had already been published in other places before appearing in the PMP Newsletter.

The director of a prison offers 100 prisoners, who have been assigned numbers from 1 to 100, a last chance at freedom. A room contains 100 boxes. The director randomly puts one prisoner's number in each closed box. The prisoners enter the room, one after another. Each prisoner may open and look into 50 boxes in any order. The boxes are closed again afterwards. If, during this search, every prisoner finds his number in one of the boxes, all prisoners are pardoned and given \$5,000 each. If just one prisoner does not find his number, all prisoners are admonished and never get another chance with this dilemma.

Before the first prisoner enters the room, the prisoners may discuss strategy — but may not communicate once the first prisoner enters to look in the boxes. There's no way for them to always win, but what is their best strategy?

checks box 50, and whether or not that's right, they continue checking boxes 51 through 99 (which would be their 50 box). If the second person finds the third person's number, they swap it with whatever is in box 50. Then the third person checks box 50, and then box 100, and then boxes 1 through 48, swapping if they find the fourth person's number. This process continues, with each person checking box 50 first, and then checking the next 49 boxes in a cycle.

In this scheme, there is a 50% chance of the first person finding the right number, but a 99% chance the second person finds the right number, and then a slightly better than 98% chance for each person thereafter (since each participant after the first looks in 49 new boxes for themselves or their successor, plus then there's a small chance that their number happened to be in box 50 without anyone putting it there). That translates into at least a $(0.5)(0.99)(0.98)^{98} > 6.8\%$ chance of all of the participants finding the right number and earning freedom, far better than the random method! And as the problem did not say anything about the participants not moving numbers, it would have to be taken as a valid method to use — truly a creative idea.

SECOND HEARING

Here's the portion of the column in which we revisit prior dilemmas, to share solutions that readers have come up with, or report on their status.

Solution to D2:

Admittedly, it looks bleak for the people in this situation. If each person involved simply checks half of the boxes at random, there's only a 50/50 chance of finding the right number, and so only a 1 in 2^{100} chance of everyone finding their number. That's a truly minuscule chance, because 2^{100} is a 30-digit number. Reader William Jones responded to this Dilemma, and emphasized how unlikely this random method is to work by correctly pointing out that “Winning the lottery is better.” William continued with a very clever idea for the prisoners involved in this situation: Each person entering the room with the boxes presumably knows the number assigned to the next one to enter. So the first one checks boxes 1-50 to look for their own number, but if they happen to see the next person's number, they swap it for whatever is in box 50. The second person first

BUT WAIT!

As it happens, though, it's possible for the people stuck in this situation to do even better than this strategy *without* moving any of the numbers. Here's the idea. Each person who enters the room starts by opening the n th box, where n is their own number. If they don't find their number, they look at whatever number was found in that box, say m , and next open the m th box. They continue in this fashion, opening the box corresponding to the number they just found in the last box, until either they find their own number or they hit their 50-box limit.

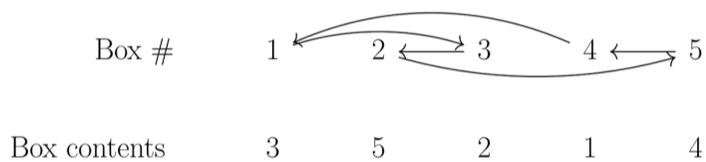


Figure 3:

How the first person might work through a few boxes, starting with Box #1

Why does this help? Imagine 100 people in a room, where each one points to one other person in a way that no two people are pointing to the same person. It has to be possible to arrange the people in a collection of "loops" where each person points to the next: just start with any person, go to the person they point to, and then the person they point to, and so on. Since there are only 100 people and everyone is pointing to someone, eventually you have to come back to the person you started with. That's your first loop. If it doesn't contain everyone, start again with a person that was left out, and create another loop. (It can't intersect with the first loop because no two people point to the same person.) Continuing this process with all of the people eventually splits everyone up into loops.

The numbers in the boxes are just like the people, if you interpret the number *in* the box as the *position* of another box. So every box is in a loop, and that loop must contain a box where the number inside is the position of the box you started with, that is, the number assigned to the person that started opening boxes.

That brings us to the kicker: if all of the loops have length 50 or less, then all of the prisoners find their own numbers! So the question becomes how likely is that? If you had 100 people and they all pointed at random, and then you divided them into loops, what are the chances that all of the loops would end up size 50 or less?

It turns out those chances are pretty good: using the combinatorics of *permutations* (as these arrangements of things all pointing to different members of their same group are called), it is possible to calculate that there is more than a $1 - \ln 2 \approx 0.307$ probability, or over a 30% chance, that the group will earn its freedom with this strategy. That's truly a long way from our initial bleak estimate of one chance in 2^{100} . ■

D3: Zero Sum Game

Contributed by Ian Stewart, University of Warwick

Is it possible to number the edges of a cube using each of the numbers -6, -5, -4, -3, -2, -1, 1, 2, 3, 4, 5, and 6 once, so that for every vertex, the sum of the numbers on the edges that meet there is zero? (See **Figure 4** below for a diagram of the cube showing where to fill in the numbers.) What about if instead you use the 12 consecutive integers from -5 to 6, inclusive?

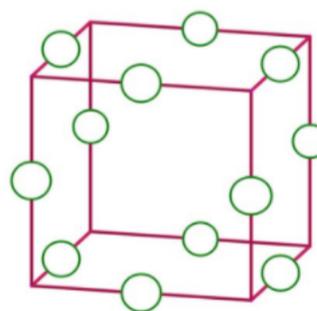


Figure 4:

Number all edges to make the sum of those meeting at each corner zero.

Solution to D3:

The Prisoner's Dilemma received a solution to the first part by reader Jesse Waite from Ft. Leavenworth, Kansas. Jesse realized that there are eight vertices in a cube, so that it's necessary to find eight sums of three of the numbers equaling zero. In addition, each number is used exactly twice in the eight sums (once for each end of the edge). That led Jesse to create the table in **Figure 5** showing a very nice pattern of three entries in each row and two in each column, so that the sums across are all zero.

-6	-5	-4	-3	-2	-1	1	2	3	4	5	6	
x							x		x			=0
	x						x	x				=0
		x				x		x				=0
			x		x				x			=0
		x		x						x		=0
	x				x						x	=0
x						x				x		=0

Figure 5: Eight sums of three numbers using -6 to -1 and 1 to 6 twice each, all equal to zero.

Armed with this pattern, Jesse was able to fit the numbers onto the edges of the cube as shown in **Figure 6**, solving the problem.

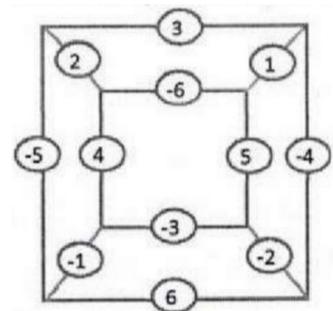


Figure 6: A cube labeled as required for the first part of Zero Sum Game

For the other part of the problem, recall that the three numbers at each vertex add up to zero. So if you add up all of the sums at all of the vertices, you still get zero. But as Jesse pointed out, adding up all of the sums adds up each number twice. So whatever numbers are used for the edges, twice the sum of all of them has to be zero. And that means the sum of all of them has to be zero. But the sum of the twelve consecutive integers from -5 to 6 is not zero, it is 6. Therefore, the task in the second half of the problem is impossible. ■

D4: Four Operations Revisited

This problem challenged you to describe the long-term behavior of four different sequences, each of which used a different one of the four basic operations on the previous two terms to determine what to add to the previous term. What is meant by "long-term behavior?" The gold standard is a closed-form formula for the sequence – an expression for the n th term that just involves n , not previous terms. For example, the closed form for the powers of two is $a_n=2^n$. If there doesn't seem to be a closed form, maybe you can find a formula that works when n is bigger than some number, but might be off for the first few values. Or sometimes you can find a formula that is never exactly right, but gets closer and closer as n gets large. All else failing, it could be one formula that's always less than the sequence and another that is always greater – and hopefully you can get those two "bounding" formulas as close to each other as possible.

All that said, here are the four sequences. Each sequence starts with 1, then 2, and thereafter adds the described quantity to the previous term to obtain the next term. If the described quantity is ever not a whole number, it should be rounded up to the next whole number.

- c_n : Add one less than the sum of the previous two terms. So to obtain the third term of this sequence, you take $1+2-1=2$ and add that to 2 to get 4.
- d_n : Add one more than the difference of the previous two terms. This time, the amount you add to get the third term is $2-1+1=2$, so the third term is again 4.
- e_n : Add half the product of the previous two terms. So in this case, our next increment is $2 \times 1 / 2 = 1$, and the third term will be 3.
- f_n : Add twice the quotient of the previous two terms. Now the next increment is $(2/1) \times 2 = 4$, and the third term will be 6.

Which of these four sequences ultimately grows the slowest?

Solution to D4:

As submitter Jesse Waite noted, a good first step is to write out the rules (known as *recurrence relations*) for the sequences in symbols. Remember that each sequence starts with 1, 2, ...

- $c_{n+2} = c_{n+1} + c_n - 1 + c_{n+1} = 2c_{n+1} + c_n - 1$
- $d_{n+2} = d_{n+1} - d_n + 1 + d_{n+1} = 2d_{n+1} - d_n + 1$
- $e_{n+2} = e_{n+1}e_n/2 + e_{n+1} = e_{n+1}(e_n + 2)/2$
- $f_{n+2} = \lceil 2f_{n+1}/f_n \rceil + f_{n+1}$

In the last sequence f , the funny brackets that only have a bar on top represent the “ceiling” operation, that rounds a (real) number that is not a whole number up to the next whole number. (Formally, we define $\lceil x \rceil$ as the least integer n such that $x \leq n$.) It might seem that we would also need the ceiling brackets in the recurrence relation for sequence e (nothing to do with the base of the natural logarithm, in this problem), but as soon as two successive entries are even, we can see the next term will be an even whole number as well, and so all of the remaining terms will be even whole numbers. Two consecutive even entries do occur, as we will see in a moment, and so the ceiling brackets are not necessary.

If none of those formulas rings a bell, a next good thing to do is to write out the first several terms of each sequence:

n	1	2	3	4	5	6	7	8
c_n	1	2	4	9	21	50	120	289
d_n	1	2	4	7	11	16	22	29
e_n	1	2	3	6	15	60	510	15810
f_n	1	2	6	12	16	19	22	25

Now let's focus on the addition-based sequence c_n as Jesse did. If a sequence still isn't looking familiar, a good third step to try is to look at its "first difference sequence," which is just the numbers you get by subtracting from each entry the one before it (you have to start from the second entry to do this). For the c_n sequence, the first differences are 2-1, 4-2, 9-4, 21-9, 50-21, 120-50, 289-120, 697-289, ...

or simply the sequence 1, 2, 5, 12, 29, 70, 169, 408,...

And now, if you have some experience with sequences, you could recognize these first differences as the *Pell sequence* p_n , a close cousin of the familiar sum recurrence (Virahanka or Fibonacci sequence), in which $p_{n+2} = 2p_{n+1} + p_n$ (instead of having $c_{n+2} = 2c_{n+1} + c_n - 1$ as in the sum recurrence). Another way you could figure this out is by taking these first differences symbolically. If we let

- $c'_n := c_{n+1} - c_n$ then,
- $c'_{n+2} = c_{n+3} - c_{n+2} = 2c_{n+2} + c_{n+1} - 1 - (2c_{n+1} + c_n - 1)$
 $= 2(c_{n+2} - c_{n+1}) + (c_{n+1} - c_n) = 2c'_{n+1} + c'_n$,

exactly the recurrence relation for the Pell sequence!



Mathematician John Pell, after whom the Pell sequence is named

So if the first differences of our sequence c_n are the Pell sequence, then our sequence must be the sum of the first $n-1$ terms of the Pell sequence (well, except possibly for an initial constant, which turns out to be one). And now you can look up or use standard linear recurrence methods to find out that the formula for p_n is

$$p_n = \frac{(1+\sqrt{2})^n - (1-\sqrt{2})^n}{2\sqrt{2}}$$

Since we are only interested in the long term behavior, and the absolute value of $1-\sqrt{2}$ is less than one, $(1-\sqrt{2})^n$ goes to zero as n grows without bound. Therefore we can ignore the second term in the numerator, and letting the constant $k=1/(2\sqrt{2})$, we have that $p_n \approx k(1+\sqrt{2})^n$.

Therefore,

$$c_n \approx k \sum_{i=0}^{n-1} (1 + \sqrt{2})^i$$

But this sum is a geometric sum, which has a convenient formula! So

$$c_n \approx k \frac{(1 + \sqrt{2})^n - 1}{1 + \sqrt{2} - 1} = \frac{(1 + \sqrt{2})^n - 1}{4}$$

This expression gives a very good picture of the long-term behavior of our first sequence c_n . (With a little more work along these same lines – don't ignore the small term in the numerator, and be careful with all of the constants – one can actually obtain an exact closed-form formula for c_n .) ■

Since each of these sequence presents its own unique puzzle, and no submissions were received for d_n , e_n , or f_n , we will accept solutions for those three sequences until the submission deadline for this issue (see right).

Submission Guidelines

Solutions to problems published in *The Prisoner's Dilemma*, and proposals for new dilemmas, are welcome. For solutions, please clearly indicated the dilemma number being solved. If a problem has multiple parts, you may submit solutions to any individual part or parts. Solutions to the dilemmas in this newsletter must be received by the deadline of 2022 September 1. Dilemma proposals will be considered on an ongoing basis. All submissions should be addressed to Glen Whitney/Prisoner's Dilemma either by email at dilemma@pmathp.org (in which case PDF format is preferred, if possible, although any reasonable format will be accepted), or by mail at:

Prison Mathematics Project
Glen Whitney/Prisoner's Dilemma
10810 N. Tatum Blvd, Suite 102-998
Phoenix, AZ 85028



Dr. Amy Shell-Gellasch

COOL MATH FROM ANOTHER TIME AND PLACE

Dr. Amy Shell-Gellasch is a full time lecturer at Eastern Michigan University. She earned her DA in mathematics from the University of Illinois at Chicago in 2000 and followed that with a post doctorate position at the United States Military Academy at West Point, NY. Her area of research is the History of Mathematics and its uses in teaching. She co-founded and currently chairs the History of Mathematics Special Interest Group of the MAA and is an associate editor of Convergence online journal. Currently she is the Chair of the Michigan Section of the MAA. She conducted research on mathematical devices at the Smithsonian National Museum of American History from 2012-2017.

Dr. Shell-Gellasch usually alternates between two columns for the PMP newsletter: *That is So Cool!* and *Mathematics from Another Time and Place*.

In this issue, her guest, Dr. Cynthia Huffman, has written a piece that falls so perfectly under both titles that we couldn't decide where it best belonged, because it includes mathematics from another time and place, but it is also VERY COOL.



Dr. Cynthia Huffman

Special Joint Issue of columns:

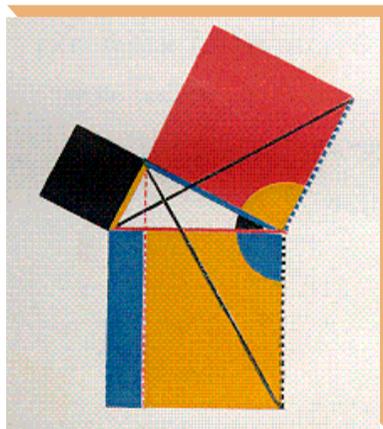
That is So Cool! and
Mathematics from Another Time and Place

by Guest Columnist
Dr. Cynthia J. Huffman
Pittsburg State University

A Presidential Proof of the Pythagorean Theorem

Ask any person on the street what their favorite theorem is from high school mathematics, or maybe ask if they can name **any** theorem from high school mathematics, and the result is likely to be " $a^2 + b^2 = c^2$." This reply is the conclusion of the Pythagorean Theorem, which more formally states that given any right triangle (that is, a triangle with a 90° angle) with legs of length a and b and hypotenuse (the long side across from the right angle) c , then the sum of the squares of the leg lengths is equal to the square of the hypotenuse. Written symbolically, the conclusion is $a^2 + b^2 = c^2$.

The Pythagorean Theorem gets its name from an ancient Greek mathematician, Pythagoras, who lived about 570 BC to 490 BC. The leader of a secret society, not much is known about Pythagoras. It is not even known if he provided a proof of the theorem himself or if it was proven by one of his disciples. In fact, there is evidence that the result was known by the Babylonians at least 1000 years before Pythagoras. From ancient times up until about the Renaissance, people viewed the Pythagorean Theorem geometrically rather than algebraically. So, when the word "square" was used in the statement of the theorem, it meant an actual square with a particular side length. Therefore, the area of a square on the



From the cover of an 1847 version of Euclid's Elements

hypotenuse of a right triangle is equal to the sum of the areas of squares on the two legs of the right triangle, like in the picture to the left.

The converse of the Pythagorean Theorem, namely if a triangle has sides a , b , and c such that $a^2 + b^2 = c^2$, then the triangle is a right triangle, is also true. The Egyptians used this fact to form right angles by using a cord with 13 evenly spaced knots, to form a 3-4-5 triangle, as in the image below.



Forming a right angle with a rope with 13 evenly spaced knots

The converse of the Pythagorean Theorem continues to be used today to form right angles. In the photograph below, it is being used with rope and stakes to lay out a swimming pool.

The Pythagorean Theorem even makes an appearance in the classic movie *The Wizard of Oz*, albeit an incorrect one. The Wizard gives the Scarecrow a brain by awarding him a Doctor of Thinkology. The Scarecrow then rattles off what sounds like the Pythagorean Theorem, but is not correct. In fact, what he states is not a theorem at all.

You can view the clip yourself at <https://www.youtube.com/watch?v=uCOxU2rKLas>



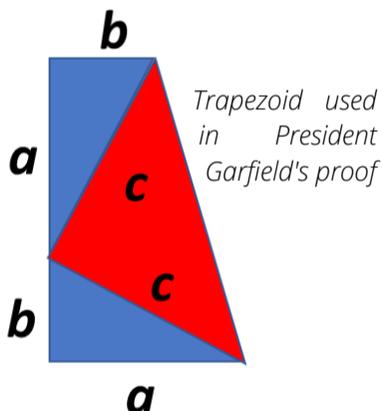
The converse of the Pythagorean Theorem being used to lay out a swimming pool.

It has been estimated that there are more than 370 different proofs of the Pythagorean Theorem, including one by Leonardo da Vinci, one by Albert Einstein when he was 12 years old, and one published by U.S. President James A. Garfield in 1876, five years before he became President. Let us take a look at President Garfield's proof.



An incorrect version appears in The Wizard of Oz.

We start off with a right triangle with sides a and b and hypotenuse c . Next form a trapezoid as in the image to the right. The area of a trapezoid is $\frac{1}{2}$ times the sum of the 2 bases times the height. For this particular trapezoid, one base has length a and the other has length b , and the height is $a + b$. Thus, the area of this trapezoid is



Trapezoid used in President Garfield's proof

$$\frac{1}{2} (a + b) (a + b) = \frac{1}{2} (a + b)^2 = \frac{1}{2} (a^2 + 2ab + b^2) = \frac{1}{2} a^2 + ab + \frac{1}{2} b^2$$

Notice that the trapezoid also consists of 2 blue right triangles, each with area $\frac{1}{2}ab$, and a red right triangle with area $\frac{1}{2}c^2$. (The blue triangle started out as a right triangle, but how do we know the red triangle is a right triangle? Well, the three angles forming the left side of the trapezoid add up to 180° where two of the angles are the two acute angles of the blue right triangle. Thus, with the two angles from blue triangles adding up to 90° , the

third one, which is in the red triangle, must be 90° , making the red triangle a right triangle.) Consequently, the area of the trapezoid is also the area of the two blue triangles plus the area of the red triangle, namely

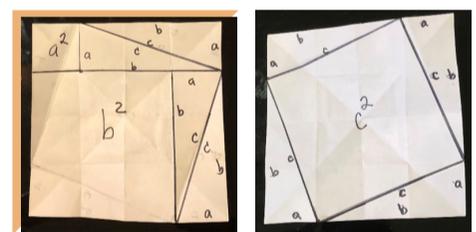
$$2(\frac{1}{2}) ab + \frac{1}{2} c^2 = ab + \frac{1}{2} c^2.$$

Now we have two different ways to express the area of the trapezoid. So, setting them equal to each other and doing a little algebra, we get the desired result and have proved the Pythagorean Theorem like President Garfield did.

$$\begin{aligned} \frac{1}{2} a^2 + ab + \frac{1}{2} b^2 &= ab + \frac{1}{2} c^2 \\ \frac{1}{2} a^2 + \frac{1}{2} b^2 &= \frac{1}{2} c^2 \\ a^2 + b^2 &= c^2 \end{aligned}$$

It is also possible to prove the Pythagorean Theorem using origami. The proof starts with a square piece of paper and, similar to the President Garfield's proof, finds the area of a region two different ways. The region in the origami proof is the starting square minus 4 congruent triangles. One way this area is seen to be 2 squares or $a^2 + b^2$, while the other way ends up with one square whose side is the hypotenuse, showing the same area to be c^2 . For a fun video by mathematician Vi Hart that walks through the origami proof, visit <https://youtu.be/z6lL83wI31E>.

Memorable and useful, the Pythagorean Theorem has been known to be true for millennia with hundreds of different proofs. Who knows? Maybe you can come up with a new and original proof of the Pythagorean Theorem to add to the list!

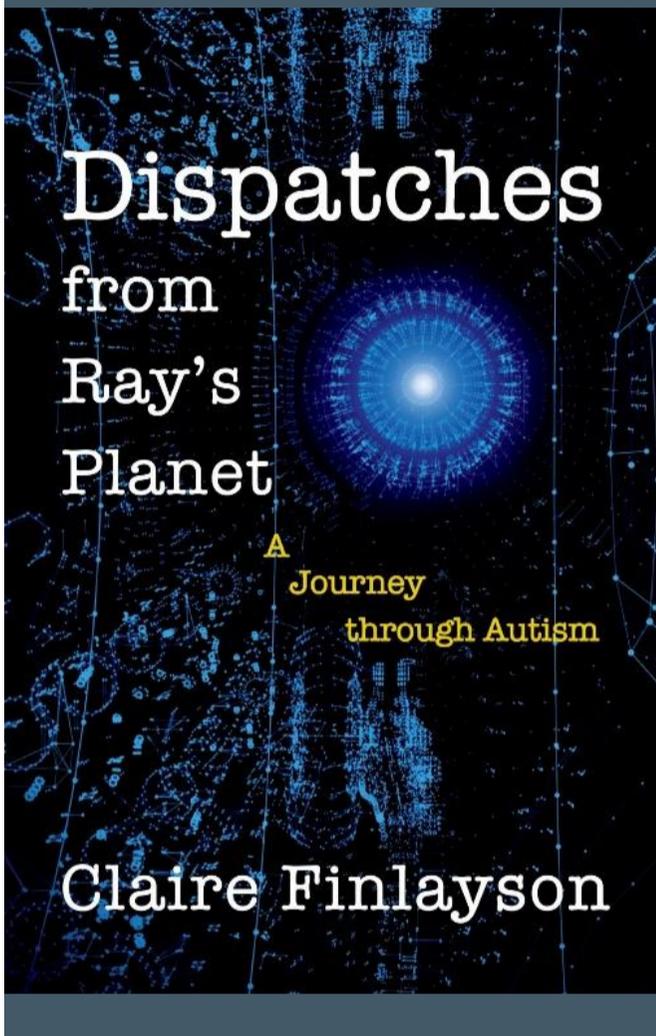


A square minus 4 congruent right triangles two different ways gives an origami proof of the Pythagorean Theorem.

If you would like to suggest a topic or submit a piece for *That is So Cool!* or *Math from Another Time and Place*, please email me at amy@pmathp.org

BOOK REVIEW REVIEW

No, it's not a typo! We actually review your book reviews!



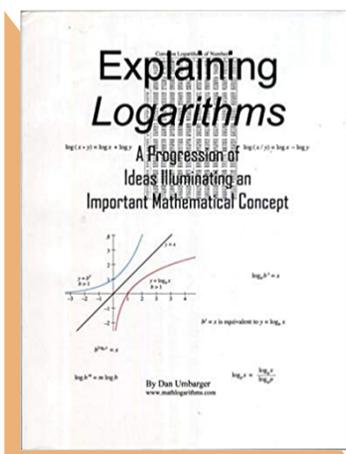
Here's how it works: You read the featured book and submit a review (details below). If your submission is deemed most inspiring to potential readers by our panel of judges, it will be printed in a future edition of the newsletter, and on the PMP website.

Our Fall 2021 featured title was, *Dispatches from Ray's Planet: A Journey through Autism*, by our very own newsletter editor, Claire Finlayson, who, she assures you, is not biased in the least when it comes to rating reviews of her book. 😊 (And neither is Ray, who is also a judge.)

Although not a math text, *Dispatches* showcases Ray's patience and dedication as a math tutor and his profound reverence for mathematics, "the language in which God wrote the universe." It also demonstrates how priceless a mentor/mentee relationship can be, and what a little confidence can do for a struggling student—all this wrapped up in a very human story told with empathy and humour from both autistic and neurotypical perspectives.

Ray is an amateur astronomer, a music and poetry lover, a night-swimmer and a free-diver. But the bedrock on which he stands is math, which never tricks him, never lets him down, and requires neither official credentials nor exquisite social

skills to access. He is a born teacher. Ray will help anyone who wants to learn—even if it means going right back to basics and drilling a kid in his times tables. *Dispatches* reached Christopher Havens in Monroe, Washington, and got passed around in the correctional complex there. That was the catalyst that led to Ray's involvement with the PMP (and Claire's, too).



Please note that there is still time for you to get in on the action by requesting a copy of *Explaining Logarithms* by Dan Umbarger, featured in the last newsletter. Umbarger, a high school math teacher, explains not only logarithms but their relationship to other core mathematical ideas. If you can't obtain the book on your own, request a copy from the PMP and submit your review for consideration by our judges and possible publication in an upcoming issue.

Reviews must be no longer than 1,000 words, and all submissions for the review of *Explaining Logarithms* should be submitted by **September 1, 2022**. See full submission guidelines are at the end of the column.

We would like to thank all those who wrote in with reviews of *Dispatches*. Our judges had a tough time choosing the winner, as each review reflected the style and personality of the writer and their individual take-aways from the book.

Here are the judges' comments:

Jon Harris wrote what would have been the perfect back jacket copy for my book. It is articulate and insightful. He relates to Ray as only a fellow spectrum-dweller can. Two thumbs up.

- C. Finlayson

Readers seem to respond viscerally and positively to *Dispatches from Ray's Planet*, universally recognizing something in themselves, or in the world around them. Daniel Josey identifies the book as a distillation of some of the research pertaining to Autism Spectrum Disorder, but he also endorses it simply as "a good story about family and overcoming obstacles." In his review Josey homes in on the challenge of switching perspective to an outsider's view. Josey selects pertinent passages from *Dispatches* that draw the reader in. He highlights examples from the book that would intrigue the prospective reader. Josey's review is a thoughtful and reasoned endorsement of the book.

Jon Harris provides lavish praise for *Dispatches from Ray's Planet*. He summarizes the challenge Finlayson took on: to try to understand her brother through the lens of Autism Spectrum Disorder. Harris's review makes us identify with Ray, the subject and protagonist. He makes us want to cheer him on in his struggles. Harris sticks his neck out with the emotional positivity of his review.

Of these two fine reviews, Jon Harris's would most tempt me to read *Dispatches from Ray's Planet*.

-D. Grunder

Each of the reviewers succeeded in making the case that *Dispatches* would reward the reader with a story at once both human and educational. I will certainly read this book now!

Of all the reviews, the one by Jon Harris had the greatest impact on me. He caught my interest early by clearly stressing that this book was definitely not an "unfeeling academic publication" on ASD or Asperger's syndrome. That interest was held when he covered some of the ways in which the book reveals how Ray's world differs from ours. Harris's review inspires the reader to imagine for a moment what navigating a Goongbalong world would be like. This reviewer succeeds in introducing us to the very human story of the journey now being taken by the siblings, Claire and Ray, and how their letters reveal the richness, the challenges and the complexities of Ray's life.

- M. Bradley

It is clear from all of the reviews I have read that Claire's writing, Ray and his story, and the evolution of their relationship touched the reviewers deeply. It's very hard to pick my favourite. I truly valued each of them. In terms of a straight book review, I think the one by Jon Harris is the most well-written, ticking almost all of the boxes I would be looking for in a review.

-J. Lussier

“ I can't possibly choose. These reviews are all deeply personal gifts and I will not rank them. That's rude on both our planets. Or should be. - Ray ”

Ray's objections notwithstanding, the judges have made their decision. The **first runner up** is...

DANIEL JOSEY, CARLISLE, IN

DISPATCHES FROM RAY'S PLANET: A Journey through Autism

Imagine living in a world where your thoughts run in constants and everything is linear and organized like math. You see things for what they truly are. No ulterior motives, no thoughts about how to elevate your social status, and nothing but good intentions. But then you find out you are actually in a world where this is not the norm at all.

This is where Ray comes in, and his sister Claire does an absolutely wonderful job at describing both sides of the spectrum.

Reading this book you learn to love Ray, all faults included, because his big heart and selflessness outweigh his social improprieties by far. "On my planet," he says, "there would be no social climbers, no subtle hints or subliminal messages to miss, and the telling of little white lies would be a capital offense." On Ray's planet, sugar-coating the truth is an insult.

Claire vowed to find Ray's planet, and if you ask me, she found it—and she explains it with meaning and feeling.

Claire didn't realize Ray might have Autism Spectrum Disorder (ASD) until Ray was 50, though after doing the research and looking back, it was pretty clear that the signs were there. On Ray's planet, the game of chess is the big game, but on Earth, it's games played without clear rules. For Ray, playing a game with unspoken rules is terrifying. Hence the new word he coined for the game of social niceties: "Goongbalong." As Ray will tell you, "I commit some Goongbalong foul that seems to entail absolute and permanent expulsion from that person's social circle. It can happen any time, which is why I'm in a state of constant terror when I'm around other people."

Because of these "G" fouls Ray commits, he is a way better communicator by email. Ray's mind lives in the non-verbal half of his head, and he feels like he has no time to process his words before "Speaking Man" takes over. He hears the words for the first time when spoken, so in retrospect a lot of people don't know how to take him.

Ray is a passionate person who has a huge heart. His actions speak volumes. From helping pull his nephew out of depression to helping anyone who will listen learn anything he's passionate about, Ray has a special quality and patience that makes him an amazing teacher. Claire's learning of Ray's ASD helped clarify so much of what was going on with Ray and what life is like on his planet. And after all Ray has dealt with—the loss of his career and being a social outcast—he comes out on top. His pros outweigh his now-better-understood cons.

If you are looking to further your knowledge about ASD, this is a highly recommended book to start with. And if you are just looking for a good story about family and overcoming obstacles—just a good read—check out *Dispatches from Ray's Planet*.

And the **winner** of the book review contest is...

JOHN HARRIS, MONROE, WA

Goongbalong: The Game of Social Minutiae

In *Dispatches From Ray's Planet*, Claire Finlayson gives a powerful and in-depth look at her struggle to understand Ray, her autistic brother. Far from the sterile and unfeeling academic publications written on Autism Spectrum Disorder, *Dispatches* is a warm, human, and realistic journey through the lives of these siblings as told by their regular correspondence.

Ray's family always knew that he was different, and these differences made their interactions with him confusing, complicated and sometimes painful. Though obviously of great intelligence, Ray struggles to understand the most basic of social concepts. These notions of common courtesies and etiquette, part of a never-ending game that Ray calls Goongbalong, seem impossibly complex and arbitrary to him. He is constantly making simple mistakes, or Goongbalong fouls, that alienate, infuriate, and bewilder those around him. It isn't until later in Ray's life when his sister picks up a book on Asperger's syndrome that things start to make sense.

As Ray has always had an easier time writing down his thoughts than he does speaking them, he and his sister begin exploring this new avenue of understanding through the writing of letters.

Ray's difficulties with those things that come naturally to others are tragic, but his triumphs are profound. From his love of math and teaching to his passion for free-diving and astronomy, Ray's life comes out in stunning detail as told by his sister and their letters.

This first book of Finlayson's is an absolute triumph. Aside from being unusually entertaining, it is heartwarming and educational. Without a doubt, *Dispatches* is a must read for anybody striving to better understand the human condition and those who fall outside of its baseline.

To submit electronically, send your review to:
PMP@pmathp.org

Mail paper submissions to:
Prison Mathematics Project
Attn: Book Review Submission
10810 N. Tatum Blvd, Suite 102-998
Phoenix, AZ 85028



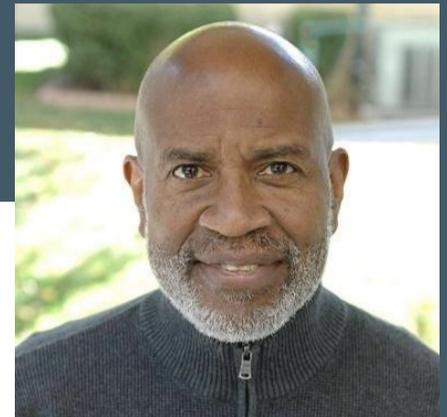
Christopher Havens

A Miscreant's Miscellany

by Christopher Havens

A Conversation on Math Inclusivity

With Lloyd Douglas



Lloyd Douglas

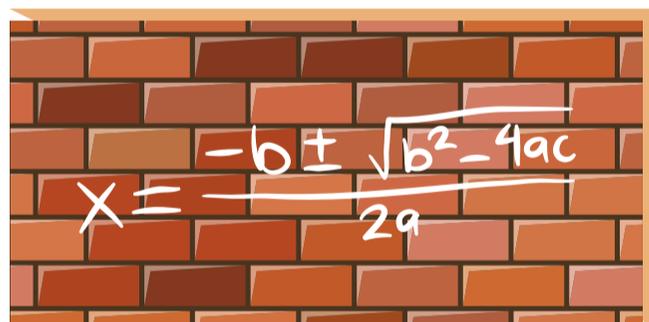
CH: Hello again readers! Today we get to meet Lloyd Douglas, a man who, like myself, works towards nurturing a love of math in people with a seed of passion. This is one of the main concepts behind what we do in the PMP! So without further delay ...Lloyd, can we begin with you telling us a little about yourself?

LD: Well, I grew up in New York City and went through the public school system there. I went to an engineering high school because I thought I wanted to be an engineer. One of the many good things about going to school there is that I learned I didn't want to be an engineer. It's better to learn these things sooner rather than later, right? Then, as a first-generation college student, I really didn't know what to expect. It was difficult for me to pick a major because I have many interests, but I ended up majoring in math. It wasn't easy. Nevertheless, I was crazy enough to try to go to graduate school for a master's degree in math—and that was even harder. I learned a lot, even though I did end up graduating.

CH: I wish I could say that I had graduated! I was a third-year freshman in high school when I tapped out, so I am still formally uneducated. Growing up, I thought math was a game played by geniuses. I had seen so many movies with brilliant cryptographers and mathematicians that they had become my idols. They were the heroes in my fantasy, slinging symbols about like wizards. But what they *did*? Well,

that was out of my reach, because of course, I am no genius.

After I began serving a 25-year prison sentence in 2010, I ended up in solitary for a year, staring at a brick wall. This is not a metaphor. I literally stood in a tiny cell staring at a brick, and seeing it for the first time as a blank slate instead of as the security against me. In that moment, I realized that with 25 years ahead, if I worked long enough and hard enough, I could maybe sling a few symbols myself.



Needless to say I'm sure your path was very different, Lloyd. Let me ask you, what first drew you to pursuing mathematics?

LD: My parents had little formal education. My father only finished sixth grade and my mother fourth grade, but they were two of the wisest people I've ever met. Everybody knows something I don't, so that means I can learn from everybody. That's one of the many lessons I've learned in life. I've had many teachers, even if teaching wasn't their formal role.

One of the things that drew me to pursuing mathematics was its ability to solve problems—and not just math problems. Everybody has problems, so that would make mathematics potentially universal. Mathematicians like to say that math is everywhere, but I don't think that even mathematicians know how true that is.

I have had good times and not-so-good times with math. One thing that kept me going was reading Albert Einstein's biography, in which he talks about failing algebra. What turned him around was changing how he looked at algebra. He figured it out and then of course went on to do greater things. I figured if Einstein could struggle with any math, then we shouldn't feel so bad about the difficulties we encounter with it. I'm no Einstein, as the expression goes, but it still gave me hope. It's not lost on me that you seeing the *brick* differently is analogous to Einstein seeing *algebra* differently.

CH: I suppose you're right! I can relate completely to your thoughts about the universality of math. For me, mathematics is a catalyst for change and a door to a world where even the most mundane things can hold within themselves true beauty. Heck, we can accomplish almost anything through the lens of mathematics, including fostering community, culture and human connection.



Okay, let's switch gears a little. You mentioned having good times and not-so-good times with math. I imagine you're not just referring to studying. Would you describe some of the greatest difficulties you've faced and then some of your triumphs?

LD: Wow! There have been so many difficulties. One of my first was plane geometry. That didn't seem at all like math to me, especially the part where I had to prove theorems. I didn't understand why I had to "prove" something that was already known. It seemed like a complete waste of time to me, and I wasn't good at trying to "unknow" things just to prove a theorem. Same thing when I took advanced calculus—more proofs of theorems that I had learned in calculus. At the time, I didn't realize most of math was proof based. Had I known that I probably would have chosen another field. Then again, I can be stubborn at times, so who knows?

Graduate school was also very difficult for many of the same reasons.

If I had to talk about triumphs, I'd start (and maybe end) with intermediate algebra and trigonometry. Being successful in those was instrumental in keeping me in math for as long as I was. But I think my greatest accomplishment in math is having been useful to others. Maybe that's not a triumph, strictly speaking, but I think that my having struggled made it easier for me to sympathize with others who also struggled with math but loved it nevertheless.

CH: I can definitely relate to the feeling that my greatest mathematical accomplishment is in helping others. It's also a struggle for me sometimes, because under my current circumstances it's hard to see progress towards the bigger picture. It's also hard for me to *know* if what I'm doing is having the desired effect ... or if I'm just being a public nuisance! But just when I'm really starting to doubt myself, I'll receive a heartfelt letter in which somebody describes how the PMP is helping them through the exact same struggles I once had.

You once said that one of your biggest goals was to give people opportunities to pursue mathematics. I wonder how the current level of inclusivity plays into your efforts. Do you see areas where we can pull together and make improvements?

LD: There are definitely areas where we can pull together and make improvements. I don't think we are inclusive enough. There are some of us who even restrict who we think can be called a mathematician. In some cultures interest in mathematics is enough to be included in that category.



In the U.S., there are often further "requirements," and even if you meet those requirements, you are told that you aren't *really* a mathematician, depending on what types of mathematics you do. That attitude certainly doesn't let more people feel as if they belong, and one of the things I've found is that inclusivity benefits the collective body even more than it does those who are being included. Many view inclusivity as an equity issue, namely that it's not fair to exclude people. While I agree that it's unfair, I think the bigger issue is that the more people we include, the better off we all are. If we exclude talented people, they will take their talent elsewhere, and wherever they go will benefit from them being there. It will be our loss.

CH: Damned right. Even in my limited ability to roam within the global mathematical community, I've encountered some folks whom I had looked up to, but who turned out to be colossal elitists. Just communicating with them was an uncomfortable experience. One of them told me that I should not have endeavored to undertake research because I lack certain degrees and the formal education behind them—but research is one of the primary ways I play and interact with math. I go exploring and it sometimes leads to neat discoveries, which I pursue. Mathematics is a practice that comes more from my heart than my mind. It's a source of so much of the beauty that surrounds me, and because of that it's sacred to me. I imagine other

people have that feeling as well, and when somebody comes along and tries to invalidate our place in mathematics, it's highly personal and discouraging. Do I consider myself a mathematician? I do. I also consider the work I do in the community to be mathematical. I know this may be hard for some people to understand, but for me, part of *doing* math involves sharing the beauty I see and inspiring people... because I believe wholeheartedly in what you say, Lloyd: the more people we include, the better off we are.

I think it's important for all of us who function in the math community to go just a little out of our way every day, in an effort to make improvements towards tackling the problems that can only be solved through conscious effort.

What are some of the things people can do that will bring us closer to the inclusive community that we all claim to strive for?

LD: We start by encouraging others. After all, people who are mathematicians were drawn to mathematics for reasons which may vary from person to person but are still reasons. Why then, would they not want to draw others into the same discipline that they love so much?

In some sense, we are *all* educators, so if I don't understand, shouldn't someone take it as their responsibility to try to help me understand if they think it's important? In almost every endeavor in life, people try to convince others of the importance of the things they think are important. Yet in mathematics there is more of a tendency to say, "I can't explain it to you because you wouldn't understand." The logic of that seems backwards to me, especially coming from people I would expect to be more logical.

CH: I agree, it is backward. I once started a program called the Social Mathematics Experiment, made up of several prisoners and a few ladies from our

educational department. Some liked math and some had huge math anxiety. But we approached things a little differently, and I got to really explore everyone's comfort levels during diverse types of teaching and learning. One thing was certain: the level of engagement and the quality of learning was so much better when the people who were farther ahead worked towards lifting the others up. There was no race to know *more* than the next person. In general, those with higher levels of mathematical maturity came to feel a responsibility to bestow their knowledge upon the others—and this actually helped both parties gain a deeper understanding by seeing each other's approach, and also by having to explain the thought processes in their own minds. You're right: we *should* encourage others and strive to help them understand things from an unfamiliar perspective. The best mathematicians are the ones who can share the beauty they see with others and help them find that beauty on their own. Besides, when others tell me about "great mathematicians," they are almost always referring to the greatness of being wonderful humans. Greatness, in the way it matters most, has nothing to do with the significance of one's discoveries.

Okay, we mentioned that there are opposing ideas as to what a mathematician even *is*. Help us out here, Lloyd. What truly makes a person a mathematician?

LD: One of the things I practice in life is that even if I think I'm right about something, I "look it up." My memory isn't always great, and sometimes I misunderstand or misinterpret what something is, so I fact-check myself.

Once during a conversation about who counts as a mathematician, I decided to look up the dictionary definition. Now, some people will argue about which dictionary should be used, etc., but my point was just to find some definition to get the conversation started, which to me is the important thing. When I looked up the definition of a

what I found was "*An expert in or student of mathematics.*"



Mathematicians come from all walks of life

One of the things people tend to do when reading—and that could be the Bible or the U.S. Constitution or anything else—is to "cherry pick." They focus on the part they like or that supports their case, and they ignore the rest. If you are going to use something as your source then you have to use it completely. So, in the definition I found, many will look at the first part and ignore the second part. But that definition includes a lot of people, and you'll note that it doesn't say "*formal* student of mathematics."

CH: Agreed. And if part of our purpose as mathematicians is to contribute to the understanding of mathematics, then by all means, we should be jumping at the opportunity to share our knowledge!

One of my closest friends defines a mathematician as someone who serves mathematics. I agree with this. It is a broad statement, which can include things like:

- Both the giving and receiving of knowledge towards a better understanding of mathematics.
- Adding to the wealth of human knowledge through various contributions to mathematics.

But adding to the wealth of human knowledge boils right down to the fact that every piece of information we put out there leads to a question answered through the giving or receiving of a better understanding of this mathematical practice of ours. The definition you found is in essence the same thing. It's not about the "credentials" of the practitioner.

Besides, mathematicians defined by their college degrees do not necessarily withstand the test of rigor. If that were the case, then for most of his life, Srinivasa Ramanujan wasn't even a mathematician. Yet, some of the mathematics he was scratching into his notebooks still baffles some of our brightest minds today! Mathematics was a part of the fabric of his being ... it was inside his heart, and *that* is what made him a mathematician long before Hardy brought him to Trinity. That's an extreme case, but there have been many other influential mathematicians throughout time who weren't even considered "mathematicians." Imagine that.

LD: I agree that sharing our knowledge should be a big goal of ours. It's true, the example of Ramanujan may be a little extreme, but it illustrates your point very well. There are also those who more than one discipline want to claim. An example of this is Sir Isaac Newton.

Srinivasa Ramanujan (left) and Sir Isaac Newton (right)



Mathematicians claim him, physicists claim him, and also he wrote religious tracts. So was he a mathematician? A physicist? A theologian? All of the above? Part of the tendency toward exclusivity is that we think people can only be one thing. Dick Zare is a former chair of the National Science Board and his research groups have been problem solvers, using whatever science is necessary to solve a problem at hand. His group's philosophy has been "There are those of us who believe that there is at most one science." That resonates greatly with me.

CH: That is a wonderful statement. Ladies and gentlemen, the inclusivity we experience in the math community is a marvelous thing, and my hat is off to Francis Su and so many others like him for

inclusivity had a direct influence on how easy it was for me to take my first steps into the social fabric of mathematics. But not everyone has such experiences, and so there are still areas we as a community can improve in, areas that are easy to take for granted, no matter how noble our intentions.

We both know that the push for inclusivity is not finished yet. And in addition to all of our continued efforts to promote this in the math community, I believe a slightly different approach should be added to our efforts, in which we become active in building awareness around various aspects of *exclusivity*. Furthermore, I think we should push this as an international topic for discussion...

Lloyd, thank you for this conversation. I know it will be the start of some remarkable discussions. Would you be so kind as to close us off with a few words to our readers?

LD: It's been a pleasure for me, Christopher. Thank you for making this all happen. Mathematics is a wonderful discipline as we both know. And there is no boundary around joy, so we don't lose any of that joy when we invite others to join us in the experience. In fact, I claim that when we include others, the total joy increases exponentially, thus making our share of joy even greater. So even from a selfish perspective, it's a good thing to do. I just ask that we give it a chance. We may be pleasantly surprised at the outcome ...



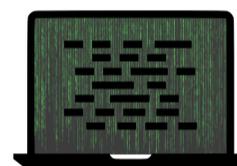
CLOSING COMMENTS

We hope you have enjoyed this fourth iteration of the fledgling PMP newsletter. As we continue to grow and evolve, we appreciate the support and encouragement you give us with your letters, book reviews, suggestions and thoughtful comments. It keeps us going. We would love to hear even more of your voices, either through mail or through the pmp@pmathp.org electronic mailbox. Drop us a line and let us know about your experience with the PMP and the impact it has had on you so far. We invite feedback from both participants and mentors. If a positive experience occurs related to mathematics, let us know! We want to be able to highlight the amazing power of higher academics in the prison system and more importantly still, the people behind it. That's you!



Christopher Havens

Announcing the NoTech Programming Software!



No-Tech Programming Input Format

```
//Name of Programming Language
//Title of Program (with no spaces)
:
Insert Program Here
:
//End
```

The Input

```
//Python
//ProductOf2x2Matrices

def matrixProduct2x2(array1, array2):
|a, b, c, d = array1[0], array1[1], array1[2], array1[3]
|e, f, g, h = array2[0], array2[1], array2[2], array2[3]
|return [a*e+b*g, a*f+b*h, c*e+d*g, c*f+d*h]

R, L, leftFactor = [1,1,0,1], [1,0,1,1], [1,0,0,1]
LRWord = [R, L, L, R, R, R, L, L, L, L]
for x in LRWord:
|leftFactor = matrixProduct2x2(leftFactor, x)

print(leftFactor)

//End
```

In math and coding news, we are currently developing a system for programming computers without using actual computers. That's right. We've developed a beta version of an ingenious NoTech Programming system that supports languages like Python and JavaScript, but without any web-dependent features. Our innovation uses the one piece of tech prisoners actually have access to: an mp4 player with a restricted email app. Our software simulates a fully automated console, allowing participants to learn and use programming to solve real-world problems in their mathematical studies. NoTech is currently in pilot testing and will launch full-scale as soon as we can secure funding.

The Output

```
Hello, this is the result of your program: ProductOf2x2Matrices!
Output:
[43, 10, 30, 7]

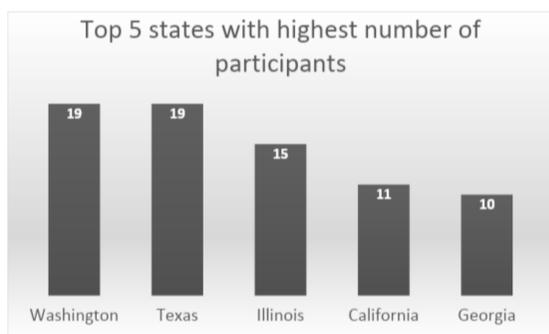
Error(s):
N/A
```

A Word to Our "In House" Participants:

Many of you are not aware that we extend our services to you after your release back into the community. For this to happen, we must know your release date. We want to keep you plugged into a mathematical lifestyle so we can help you continue to flourish! And even if you're not going to be pursuing mathematics post-release, we still want to keep in touch. So if you haven't already done so, please contact us and let us know your release date.

Many of you will have received our survey. It's quick and simple to complete. It's very important that we start collecting baseline information and measuring our progress, so please make sure to send it back.

Relevant Numbers at a Glance:



Total number of mentor/mentee pairings	171
Total number of volunteers	466
Total number of facilities in which PMP is active	82
Total number of states in which PMP is active	27

Start A Math Circle!

Part of what we aim to do is shift the current culture of prison into one that focuses on the pursuit of higher academics. One way this can be done is through the implementation of math circles.

To Our Supporters, Donors, and Stakeholders:

We have reached a point where long-term sustainability is on the horizon, and we are excited to share some key milestones with you. But first, some important insights.

How we define success:

The PMP's success is measured by desistance from crime and antisocial behaviour among our participants post-release, along with healthy community involvement and positive contributions to society. Our pathway to success is through direct mentorship by members of the math community who work one-on-one with our participants. The way we grow our successes is by:

- Increasing the number of quality mentor/participant pairings
- Equipping our participants to be culturally impactful
- Expanding into additional academic subjects, such as science, technology, engineering, literature and art

While we want to increase our footprint, we want to increase it in a way that is effective, in accordance with our mission and values.

As you can see, Washington and Texas lead the way in the number of participants. It is worth noting that 89% of Washington's participants are concentrated in one facility. This facility has seen an increase in participation of 112.5% over six months, due mainly to two factors:

1. **The introduction of informal math circles**
2. **The cultural influence of current participants**



A math circle is a learning space where participants engage in the depths and intricacies of mathematical thinking, propagate the culture of mathematics and create knowledge.

To reach these goals, participants partake in problem-solving, mathematical modeling, art and philosophical discourse.

Many of you are already forming circles (we're tipping our hats to you, Washington and Georgia) and we need more of them. Here are some tips on how to conduct a math circle in prison:

- Hold your circles weekly and at consistent times
- Choose a facilitator/host who can set a theme or promote conversation around a topic
- Adjust for all levels of math, including beginners
- Have fun!

This is an exciting time for us as we are fine-tuning the details on our strategic plan for the next three years. This roadmap will offer guidance on how to reach a disenfranchised community with meaningful support that is truly effective at reducing recidivism while shifting the prison culture to make an even bigger impact. The PMP isn't about giving lip service to the cause: we aren't saying we are reaching "x" number of people and then not really helping them. That would be like a major corporation "greenwashing" the public by saying they are environmentally friendly while not really doing anything to help the environment. When it comes to rehabilitation and prisoner support we do not want to fall into the trap of "support-washing." We want to do it for real.

We are grateful to our incredible supporters who stand behind what we do. We would not have been able to make it this far without your help and heart. You have seen the impact our program has already had in people's lives, and you know that further positive change is both possible and realistic. Your support could help hundreds of thousands of prisoners learn how to use the most valuable tool in our modern era.

Of course you will need to make adaptations so this works for your specific environment.

A few things to keep in mind:

- Math circles do not have to be *just* math. They can encompass any subject that focuses on an academic concept.
- Higher education must transcend prison politics. A standard should be set to be inclusive—and by that we mean *fully* inclusive.
- Be creative! If you need support, we are here to provide it. Let's collaborate!

On the community side of things, we are seeking a few dedicated volunteers for key roles, including:

- **Fundraiser and/or grant writer**
- **Social Media Marketer/Influencer/Content Creator**

To inquire about either of these roles please contact our community liaison, Ruth Utnage at ruth@pmathp.org and reference the position in the subject.

We cannot wait to have future successes with you!

Wishing you all the best until the next iteration.

Christopher Robin Havens
Executive Troublemaker
Prison Mathematics Project



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